

## 3D Multi-static SAR System for Terrain Imaging Based on Indirect GPS Signals

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**Abstract.** A 3D multi-static SAR imaging system which utilises reflected GPS signals from objects on the Earth's surface is described in this paper. The principle of bistatic radar is used to detect movement of, or changes to, the imaged object. The indirect GPS signals are processed by a match filter with the aim of improving the spatial resolution of detection. The measure of spatial resolution of this imaging system is derived, and is confirmed by MATLAB simulation. Several scenarios are considered, for the visible satellite at a given receiver and object location. The scenarios for different satellites are: a) static receiver with two targets which move with the same speed; and b) moving receiver with one static target and one moving target. Simulation results show that the spatial resolution of detection depends on the relative positions of the GPS satellites, the imaged objects and the GPS receiver, as well as their respective velocities.

**Key words:** Detection, Imaging, GPS, SAR

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### 1 Introduction

The Global Positioning System (GPS) is an all-weather, global, satellite-based, round-the-clock Global Navigation Satellite System (GNSS). Measurements on the direct GPS signals have been successfully used in navigation and positioning, while indirect or reflected signals are viewed as a nuisance. However, scattered/reflected GPS signals also can be 'reused' for remote sensing, radar target detection and (reflector)

change detection. Examples of remote sensing applications are ocean altimetry, wind speed/direction determination, monitoring of sea ice condition, and for the determination of soil moisture content [1~7]. Analysis of indirect GPS signals has recently attracted a lot of attention because of its potential civilian/military applications. [8~13] established the models for the extraction of sea state and wind speed from ocean reflected GPS signals, and carried out some reflected GPS experiments. A parallel delay mapping GPS receiver on an aircraft was used to confirm the modelling. [14] made ocean altimetry measurements using reflected GPS signals observed from a low-altitude aircraft.

The irradiated power of GPS satellites can also be reused for imaging, based only on the analysis of indirect GPS signals. Generalising the bistatic radar concept, this paper describes a multi-static synthetic aperture radar (SAR) system consisting of a constellation of visible GPS satellite transmitters, a multi-channel modified GPS receiver and multiple objects. The 'objects' may be one or more moving platforms such as a ship, or a reflective surface that is monitored for its movement. This imaging system has the following useful properties: (a) no dedicated signal transmitter is required; (b) the GPS signal frequency is reused; (c) GPS operates round-the-clock and its signals cover the entire Earth's surface; (d) low power consumption; and (e) known GPS signal structure. That is, the multi-static SAR system has the potential to develop high quality, and low cost images, of a localised area.

## 2 3D Multi-static SAR System Model

The terrain imaging system provides visual discrimination within the image scene. A common measurement of the ability for spatial discrimination between objects is the spatial resolution. [15] has described the resolution equations for a 2D configuration in which transmitter trajectory, imaged object, as well as receiver are in the same plane. The bistatic SAR principle is traditionally based on the radar positioned on an airborne platform. The transmitter and receiver are on the same moving platform while the imaged object is static. When the radar moves, the reflected signal from the same imaged object is processed in order to synthesise an antenna with a synthetic aperture.

In this paper, a 3D multi-static SAR system, as illustrated in Figure 1, is set up as a terrain imaging system, where transmitter and receiver are located in two separate positions and move with different speeds. There could be one or more imaged objects with different velocity vectors. The visible GPS satellites ( $Tr$ ) at the receiver position ( $R$ ) act as a series of continuous signal transmission sources. The  $i$ -th visible GPS satellite  $Tr_i$  moves with velocity  $\mathbf{V}_{Tr_i}$ .  $\mathbf{V}_{O_j}$  and  $\mathbf{V}_R$  express the velocity vectors of the  $j$ -th imaged object  $O_j$  and the GPS receiver  $R$  respectively.  $R$  is placed near the object  $O_j$ . All coordinates of  $Tr_i$ ,  $O_j$ , and  $R$ , as well as their velocity vectors, are expressed in the Earth Centred Earth Fixed (ECEF) coordinate system.

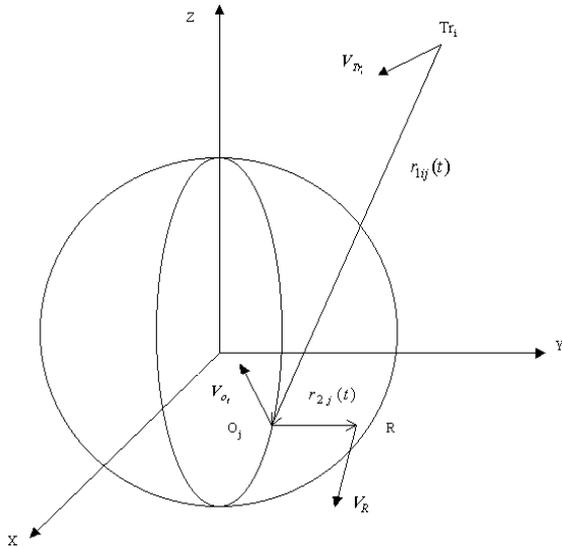


Fig. 1 3D Multi-static SAR System Model

For the terrain imaging application, the bistatic radar principle and the synthetic aperture radar technique are used in the signal processing. If  $Tr_i$ ,  $O_j$ , and  $R$  move with constant velocities during the observation period, the signal obtained at the  $O_j$  position will be a linear

frequency modulated (FM) GPS signal. The received indirect GPS signal at receiver  $R$  will be an approximately linear FM GPS signal. It has been demonstrated that the range resolution of radar is an inverse ratio of the bandwidth of the signal. Hence this makes it possible for the multi-static SAR imaging system to get an enough acting range and resolution simultaneously, since the linear frequency modulated GPS signal has the good property of pulse compression.

## 3 Imaging Resolution

In such a multi-static SAR imaging system, indirect GPS signals that have been reflected from objects are used for their detection using the bistatic radar principle. The spatial resolution is enhanced by the synthetic aperture radar (SAR) technique.

As shown in Figure 1, suppose the ranges between  $Tr_i$  and  $O_j$  and between  $O_j$  and  $R$  at the beginning of the observation period are represented by  $\mathbf{R}_{1ij}$  and  $\mathbf{R}_{2j}$  respectively. During the period of measurement, the corresponding ranges are varied with respect to time  $t$  and are represented by  $r_{1ij}(t)$  and  $r_{2j}(t)$  respectively.

$$r_{1ij}(t) = \left| \mathbf{R}_{1ij} + (\mathbf{V}_{Tr_i} - \mathbf{V}_{O_j}) \cdot t \right| \quad (1a)$$

$$r_{2j}(t) = \left| \mathbf{R}_{2j} + (\mathbf{V}_R - \mathbf{V}_{O_j}) \cdot t \right| \quad (1b)$$

Suppose the signal which is transmitted from the GPS satellite is:

$$S_{Tr_i}(t) = d(t) \cdot \text{Re}[A \cdot \exp(jw_c t)] \quad (2)$$

where  $d(t)$  is the C/A code (or P code), and  $w_c$  is the carrier frequency. The received signal at  $O_j$  in complex form is:

$$S_{O_j}(t) = d[t - \alpha_{1ij}(t)] \cdot K_{1ij} \cdot F_{1ij} \cdot A \cdot \exp\{jw_c \cdot [t - \alpha_{1ij}(t)]\} \quad (3)$$

where  $\alpha_{1ij}(t) = r_{1ij}(t)/c$  is the time delay of the signal from the GPS satellite  $Tr_i$  to object  $O_j$ ,  $c$  is the speed of light, and  $F_{1ij}$  is the scatter coefficient.  $K_{1ij}$  is a factor which is associated with  $r_{1ij}(t)$  and  $F_{1ij}$ .

The Doppler frequency shift caused by the movement of  $Tr_i$  and  $O_j$  is:

$$f_{1ij}(t) \approx -\frac{1}{\lambda} \cdot \frac{1}{R_{1ij}} \left[ \left| \mathbf{V}_{Tr_i} - \mathbf{V}_{O_j} \right|^2 \cdot t + R_{1ij} \cdot \left| \mathbf{V}_{Tr_i} - \mathbf{V}_{O_j} \right| \cdot \cos \gamma_{Tr_i} \right] \quad (4)$$

(if  $R_{1ij} \gg \left| \mathbf{V}_{Tr_i} - \mathbf{V}_{O_j} \right| \cdot t$ )

where  $\gamma_{Tr_i}$  is the angle between  $\mathbf{V}_{Tr_i}$  and  $V_{O_j}$ , and  $\lambda$  is the wavelength of the GPS signal. Then R receives the reflected GPS signal from object  $O_j$ :

$$S_{R_j}(t) = d \left[ t - \alpha_{1ij}(t) - \alpha_{2j}(t) \right] \cdot K_{2j} \cdot F_{2j} \cdot A \cdot \exp \left\{ j w_{O_{ij}}(t) \cdot \left[ t - \alpha_{2j}(t) \right] \right\} \quad (5)$$

where  $\alpha_{2j}(t) = r_{2j}(t)/c$  and  $w_{O_{ij}}(t) = 2\pi \cdot [f_c + f_{1ij}(t)]$ .

The Doppler frequency shift caused by the relative movement of  $O_j$  and R is:

$$f_{2ij}(t) = -\frac{1}{2\pi} \cdot \frac{d}{dt} \left[ w_{O_{ij}}(t) \cdot \frac{r_{2j}(t)}{c} \right] \quad (6)$$

where  $\gamma_{R_j}$  is the angle between  $V_{O_j}$  and  $V_R$ .

The frequency at R is:

$$f_{R_j}(t) = f_c + f_{1ij}(t) + f_{2ij}(t) \quad (7)$$

Thus, the received signal at R is:

$$S_{R_j}(t) = d \left[ t - \alpha_{1ij}(t) - \alpha_{2j}(t) \right] \cdot K_{2j} \cdot F_{2j} \cdot A \cdot \exp \left[ j 2\pi \int_0^t f_{R_j}(t_1) \cdot dt \right] \quad (8)$$

As the coefficients of items with  $t^2$  can be ignored,  $S_{R_j}(t)$  can be considered as an approximately linear FM signal. In order to improve the bearing resolution,  $S_{R_j}(t)$  must be compressed. Because the auto-correlating function of a linear FM signal exhibits a narrow pulse property, the output wave will become even more narrow when  $S_{R_j}(t)$  is passed through a matching filter. (A matching filter is also an optimum filter for signal detection in a white noise environment.) Hence, a matching filter is employed in the received signal processing to improve the system resolution. The output of the matching filter is:

$$S_{ij}(\tau) = \int_{\frac{T_s}{2}}^{\frac{T_s}{2} - \tau} S_{R_j}(t + \tau) \cdot S_{R_j}^*(t) dt \quad (9)$$

where  $T_s$  is the observation time. It can be demonstrated that the normalised envelope  $E_{nv}[\bullet]$  of  $S_{ij}(\tau)$  is:

$$E_{nv} [S_{ij}(\tau)] = \int_{\frac{T_s}{2}}^{\frac{T_s}{2} - \tau} \exp(jk_{aij} \cdot t \cdot \tau) \cdot dt = \sin c \left[ \frac{1}{2} \cdot k_{aij} \cdot \tau (T_s - |\tau|) \right] \quad (10)$$

where

$$k_{aij} \approx -\frac{2\pi}{\lambda \cdot R_{1ij}} \cdot |\mathbf{V}_{Tr_i} - \mathbf{V}_{O_j}|^2 - \frac{2\pi}{\lambda \cdot R_{2j}} \cdot |\mathbf{V}_R - \mathbf{V}_{O_j}|^2 \quad (11)$$

Since the characteristic of the correlation function near the peak of the compressed wave is of interest in this discussion of resolution, let  $T_s \gg |\tau|$ . So:

$$E_{nv} [S_{ij}(\tau)] \approx \sin c [k_{aij} \cdot \tau \cdot T_s / 2] \quad (12)$$

The spatial resolution in directions  $x$  and  $y$  is:

$$\rho_x \approx \frac{\lambda}{T_s} \frac{\frac{|V_{Tr_x} - V_{O_jx}| + |V_{Rx} - V_{O_jx}|}{|\mathbf{V}_{Tr_i} - \mathbf{V}_{O_j}|^2} + \frac{|V_R - V_{O_j}|^2}{R_{1ij} + R_{2j}}}{R_{1ij} + R_{2j}} \quad (13a)$$

$$\rho_y \approx \frac{\lambda}{T_s} \frac{\frac{|V_{Tr_y} - V_{O_jy}| + |V_{Ry} - V_{O_jy}|}{|\mathbf{V}_{Tr_i} - \mathbf{V}_{O_j}|^2} + \frac{|V_R - V_{O_j}|^2}{R_{1ij} + R_{2j}}}{R_{1ij} + R_{2j}} \quad (13b)$$

The authors have simulated such an imaging system, with the transmitters, receiver, and objects moving at different velocities.

#### 4 Simulations

Assume that the observation time is from 00:00:00 24 Sept. 2001 to 00:00:02 24 Sept. 2001. The coordinate of receiver R is  $[x_R, y_R, z_R] = [-4641138, 2555708, -3539132]$ m. At this time the distribution of visible GPS satellites is illustrated in Figure 2. As an example, the signals from satellites #2 and #4 are used in the object imaging, and their characteristics are shown in Table 1. Suppose there are two targets ( $O_j$ ,  $j=1,2$ ) in initial locations  $[x_{O_1}, y_{O_1}, z_{O_1}] = [-4643559, 2557041, -3534962]$ m and  $[x_{O_2}, y_{O_2}, z_{O_2}] = [-4643607, 2557068, -3534879]$ m respectively.

Tab. 1 Position & Velocity Parameters of Satellite #2 and #4 During Observation Time

GPS Sat. No.	#2	#4
Position [ $x_{Tr}$ , $y_{Tr}$ , $z_{Tr}$ ] $10^6$ m	(-25.15, -8.04, -4.15)	(-23.57, 12.33, -1.90)
Velocity ( $V_{Trx}$ , $V_{Try}$ , $V_{Trz}$ ) $10^2$ m/s	(-4.05, -5.41, 30.23)	(1.07, -3.12, -31.83)

Figure 3 shows the simulation results for each scenario in Table 2. Figure 3(a)(b) and (c)(d) show that the resolution  $\rho$  is related to the relative velocities of  $O_j$  and R. For a static receiver and moving target with speed (0,10,0)m/s, its spatial resolution is the same as that for a static target

with receiver moving with velocity (0,-10,0)m/s. The greater the relative speed, the higher the resolution. As

indicated in Figure 3(a)(c) and (b)(d),  $\rho$  is also a function of the position of Tri with respect to R and Oj.

0.0 Hours of Coverage from 9/24/00 0:00:00 for GPS Visibility for Boulder, CO

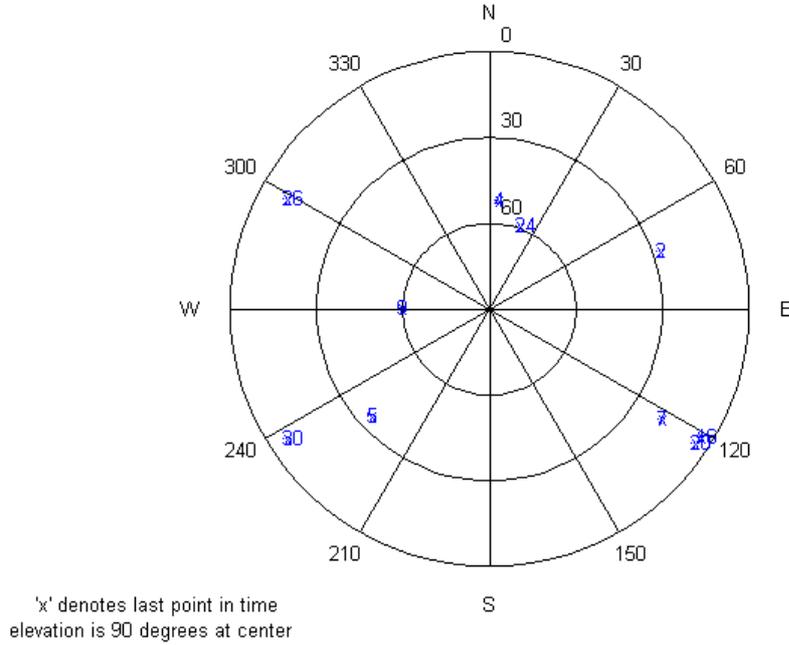


Fig. 2 Distribution of Satellites During Observation Time

Tab. 2 Scenarios in the Simulation

GPS Sat. No.	$(V_{Rx}, V_{Ry}, V_{Rz})$ m/s	$(V_{O1x}, V_{O1y}, V_{O1z})$ m/s	$(V_{O2x}, V_{O2y}, V_{O2z})$ m/s	object 1		object 2	
				$\rho_{o1x}(m)$	$\rho_{o1y}(m)$	$\rho_{o2x}(m)$	$\rho_{o2y}(m)$
2	(0,0,0)	(0,10,0)	(0,10,0)	113	154	113	154
2	(0,-10,0)	(0,0,0)	(0,7,0)	113	152	105	142
4	(0,0,0)	(0,10,0)	(0,10,0)	26	79	26	79
4	(0,-10,0)	(0,0,0)	(0,7,0)	26	76	24	73

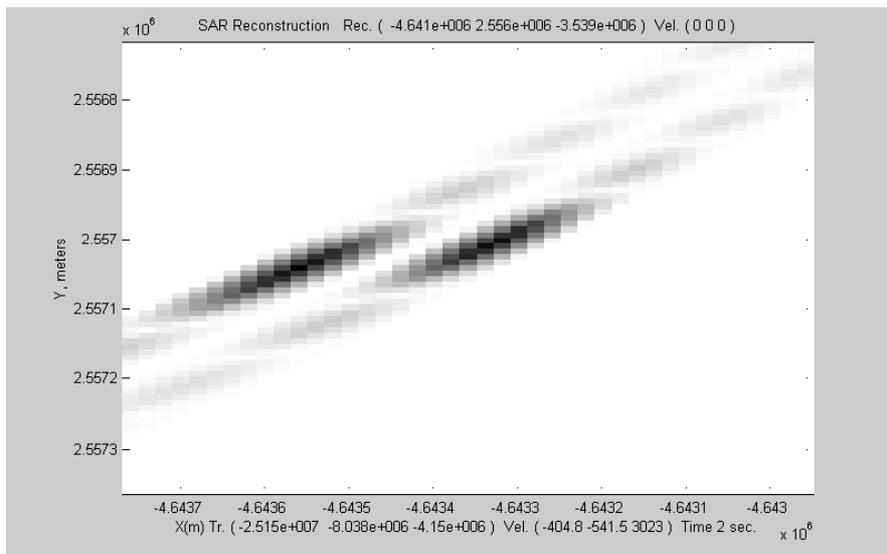


Fig. 3: Simulation Result

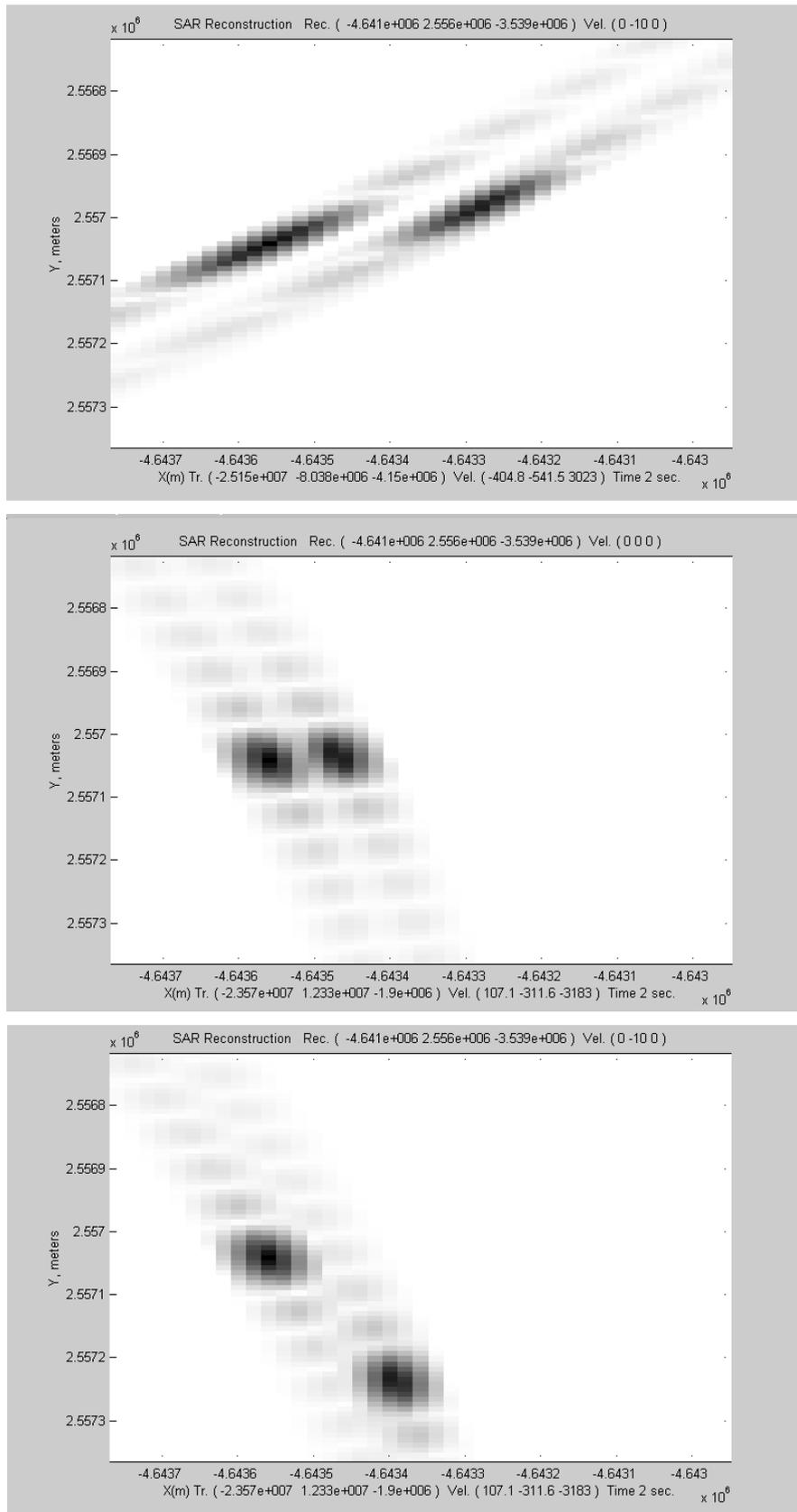


Fig. 3 Simulation Result (Continuous)

## 5 Concluding Remarks

This paper describes a 3D multi-static SAR imaging system using the reflected GPS signals. The bistatic radar principle is used to detect movement of, or changes to, the imaged object. The indirect GPS signals are processed by a match filter with the aim of improving the spatial resolution of detection. The measure of spatial resolution of this imaging system is derived, and is confirmed via simulation studies.

The simulation results show that the indirect GPS signals can be used for certain remote sensing applications. In such multi-static SAR imaging system the detection based only on the reflected GPS signal can be made for moving objects and a moving receiver. The spatial resolution is a function of the mutual positions and velocities of the satellites, imaged objects, and receiver. The 3-D multi-static SAR model also has potential benefits in sea surface imaging and target detection.

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