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Fractional Kalman filtering model on data of monitoring accelerometer deformation

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Abstract: The deformed or vibratory behaviors will exceed the threshold of building under the influence of external factors, so that it is necessary to monitor the variety of deformed body. Accelerometer is widely used in deformation monitoring due to small size and high sampling rate. In this paper, the fractional Kalman filter is introduced to update the accelerometer data. The influence of the order of different fractional derivatives on the filtering results of the accelerometer is studied and compared. The results show that when the system noise and measurement noise are fixed, using different derivative orders and comparing the filtering results under different derivative orders, the root mean square error of the fractional filtering model is smaller. Compare the filtering results under different noise variances. By comparing the errors of the two models, the image shows that the fractional Kalman filter model has better filtering performance than the standard Kalman filter model.

Keywords: Accelerometers, Deformation monitoring, Fractional model, Kalman filtering.

1. Introduction

The deformed phenomenon widely exists in many different disciplines and engineering fields. Deformation data cannot be accurately described because of its complexity and irregularity[1,2]. There is a certain correlation between these data. The deformation of the building cannot be avoided. The deformed body is usually in a state of equilibrium. But the internal structure of the building will change under the influence of extreme external environment, leading to the abnormal state of the building, which will cause continuous damage to the building structure[3]. Accelerometer is widely used in structural monitoring, which has the advantages of small size, light weight, high sampling rate and so on. It is sensitive to higher frequency deformation information, but it is difficult to detect lower frequency quasi-static deformation information[4]. The deformation law is studied and predicted through long-term monitoring results[5,6].

As a classical filtering algorithm, Kalman filter is widely used in various theoretical research and engineering[7]. The standard Kalman filter is easy to cause update interruption and divergence of filtering results when removing abnormal observation information. Some basic problems need to be solved urgently in control theory and practical engineering, such as observation and controller design. In recent years, it has been observed that fractional derivative is more accurate to describe certain systems[9, 10] with the rapid development of fractional systems[8], such as rheological model[11], chaotic model[12] and fractal model. Therefore, fractional derivative is introduced into the existing Kalman model to reduce the influence of noise on the position measurement error of accelerometer and obtain higher accuracy data[13].

Accelerometers are usually applied in actual deformation monitoring of building bodies. The fractional Kalman filter model is established in this paper in order to process the data obtained from accelerometers preferably. Through the processing and analysis of the deformation monitoring data of the two models, it is proved that the fractional Kalman filtering model can get better filtering results under the appropriate order by comparing with the standard Kalman filtering model.

2. Fractional Kalman filtering model

2.1 Kalman filtering

Kalman filter is a state vector estimation algorithm based on a set of observed values and discrete model information. The discrete model of the system is generally described by differential equation:

$$\frac{dx_k}{dt} = Ax_k + Bu_k \tag{1}$$

The subscript k represents the time corresponding to t_k. A linear discrete state equation can be obtained by solving the differential equation and discretizing it. For the standard Kalman filter model, the following discrete linear system can be established:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{2}$$

$$y_k = Cx_k + v_k \tag{3}$$

Where x_k is the state vector, u_k is the system input, w_k is the system noise, v_k is the measurement noise, and y_k is the observation equation.

The optimal estimation of Kalman filtering can be obtained from the following set of recursive formulas.

$$\begin{aligned} \tilde{x}_{k+1} &= A\hat{x}_k + Bu_k \\ \tilde{P}_k &= AP_{k-1}A^T + Q_{k+1} \\ \hat{x}_k &= \tilde{x}_k + K_k\left(y_k - C\tilde{x}_k\right) \\ P_k &= \left(I - K_kC\right)\tilde{P}_k \\ K_k &= \tilde{P}_kC^T\left(C\tilde{P}_kC^T + R_k\right)^{-1} \end{aligned} \tag{4}$$

Among them, $\tilde{P}_k$ and P_k are the prior and posterior covariance matrices, respectively; $\tilde{x}_k$, $\hat{x}_k$ are prior and posterior estimates, respectively; K_k is the Kalman gain.

When the state of the previous moment is known, the prior estimated value can be modified by the observed value y to obtain the state estimation of t_k.

2.2 Fractional Kalman filtering

In the process of discretization of the standard Kalman filter state model, The fractional G-L difference is given by[13]:

$$\Delta^n x_k = \left(h^n\right)^{-1}\sum_{j=0}^{k}(-1)^j\binom{n}{j}x_{k-j} \tag{5}$$

Where $n \in R$ is the order of fractional difference, h is the sampling time interval, k is the number of samples for calculating derivative, The factor $(n; j)$ can be obtained from:

$$\binom{n}{j} = \begin{cases} 1 & j = 0 \\ \dfrac{n(n-1)\ldots(n-j+1)}{j!} & j > 0 \end{cases} \tag{6}$$

In actual model building, there will be multiple equations, and different orders can be taken for different equations. For cases where the order of the equations is different, that is, when dealing with equation system problems, the generalized definition is given by[13]:

$$\begin{aligned} \Delta^\gamma x_{k+1} &= A_d x_k + Bu_k + w_k \\ x_{k+1} &= \Delta^\gamma x_{k+1} - \sum_{j=0}^{k}(-1)\gamma_k x_{k-j} \\ y_k &= Cx_k + v_k \end{aligned} \tag{7}$$

where $\boldsymbol{A}_d = \boldsymbol{A} - \boldsymbol{I}$ (where $\boldsymbol{I}$ is the identity matrix), and

$$\gamma_k = \mathrm{diag}\left[\binom{n_1}{k} \quad \ldots \quad \binom{n_N}{k}\right] \tag{8}$$

$$\Delta^\gamma x_{k+1} = \begin{bmatrix} \Delta^{n_1} x_{1,k+1} \\ \ldots \\ \Delta^{n_N} x_{N,k+1} \end{bmatrix} \tag{9}$$

n_1, n_2,..., n_N is the order of the state equation, and N is the number of state equations. The covariance and optimal estimation can be obtained to establish a fractional Kalman filtering model by using a calculation method, which is similar to traditional Kalman filtering models:

$$\Delta^{\gamma}\tilde{x}_{k+1} = A_d\hat{x}_k + Bu_k + w_k$$
$$\tilde{x}_{k+1} = \Delta^{\gamma}\tilde{x}_{k+1} - \sum_{j=0}^{k}(-1)^j\gamma_k\hat{x}_{k-j}$$
$$\tilde{P}_k = (A_d + \gamma_1)P_{k-1}(A_d + \gamma_1)^T + Q_{k+1} + \sum_{j=2}^{k}\gamma_j P_{k-j}\gamma_j^{\;T} \tag{10}$$
$$\hat{x}_k = \tilde{x}_k + K_k(y_k - C\tilde{x}_k)$$
$$P_k = (I - K_k C)\tilde{P}_k$$

where

$$K_k = \tilde{P}_k C^T\left(C\tilde{P}_k C^T + R_k\right)^{-1} \tag{11}$$

with the initial conditions

$$x_0 \in R^N, P_0 = \mathrm{E}\left[(\tilde{x}_0 - x_0)(\tilde{x}_0 - x_0)^T\right]$$

3. Construction of accelerometer filtering model

The Kalman filtering model is a continuous correction and prediction process that can predict dynamic systems. The state space equation of the model during the measurement process can be expressed as[14]:

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}\tilde{a} + w \tag{12}$$

In the equation, x, $\dot{x}$, $\ddot{x}$ are defined as displacement velocity acceleration respectively, $\tilde{a}$ represents measured acceleration, and w is the system noise, η_a is the system noise, $\eta_a(k) \sim N(0,q)$, q is the variance of acceleration.

The model observation equation can be expressed as:

$$z = x_m = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} + v \tag{13}$$

where x_m represents the measurement displacement and v represents the measurement noise, $\eta_d(k) \sim N(0,r)$, r is the displacement variance.

According to equations (12) and (13), the state model of a two-dimensional accelerometer can be represented:

$$\begin{bmatrix}x_{k+1}\\ \dot{x}_{k+1}\end{bmatrix} = \begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_k\\ \dot{x}_k\end{bmatrix} + \begin{bmatrix}\frac{T^2}{2}\\ T\end{bmatrix}\tilde{a}(k) + w(k) \tag{14}$$

$$z(k) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_k\\ x_k\end{bmatrix} + v(k) \tag{15}$$

wherein x_k represents the displacement at kT, $\dot{x}_k$ represents the velocity at kT, $z(k)$ represents the observation results at kT, $w(k)$ represents the system noise at the kT time, $v(k)$ represents the measurement noise at kT, and T is the sampling interval of the accelerometer.

The accelerometer is the only noise source in state time updates, whereas the process noise matrix Q and measurement noise matrix R of Kalman filtering can be constructed through the law of error propagation:

$$w(k) = \begin{bmatrix}\frac{T^2}{2}\\ T\end{bmatrix}\eta_a(k);\; Q = q\begin{bmatrix}\frac{T^3}{3} & \frac{T^2}{2}\\ \frac{T^2}{2} & 1\end{bmatrix} \tag{16}$$
$$v(k) = \eta_d(k);\, R = \frac{r}{T}$$

4. Data processing

The following introduces the processing route of fractional Kalman filtering. Calculating velocity and displacement information based on accelerometer data, establishing a discrete model of fractional Kalman filtering based on velocity and displacement information, and updating the established filtering model to obtain the updated accelerometer data. The obtained accelerometer data are used for the next filtering.

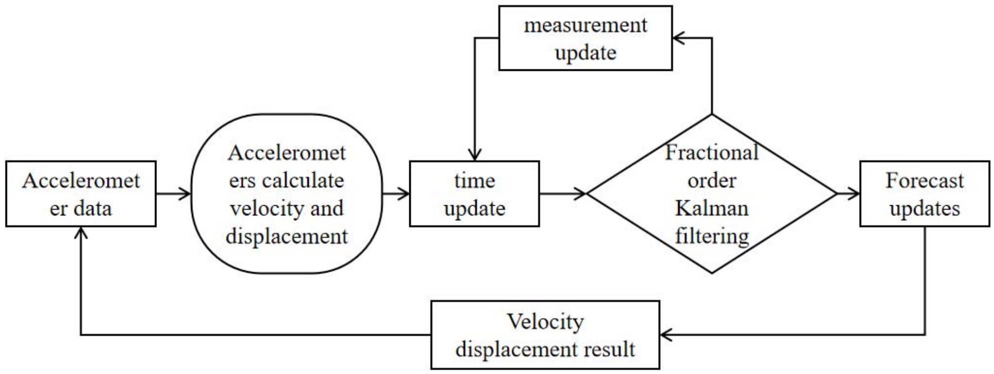

Figure 1: Flow chart of accelerometer fusion Kalman filtering algorithm

It is inevitable to be affected by various factors in the process of high precision measurement, such as light intensity and temperature. The data may contain a lot of random noise, which will have a great impact on deformation prediction and analysis. In this paper, by analyzing the characteristics of the model deformation data, fractional Kalman filter is used to denoise the observed data, eliminate the random noise existing in the observed data, and make the observed data closer to the real data, thus improving the accuracy of the data.

Calculate the root mean square error (RMSE)

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{n}\left[\hat{x}_k - x_k\right]^2} \tag{19}$$

between the filtered data and the real data to evaluate the denoising effect of fractional Kalman filtering. Usually, the smaller the RMSE value, the better the denoising effect.

5. Case analysis

5.1 Fractional order comparison

We test the accelerometer model according to the model established in Section 3. When the fractional order is different, the state equation of fractional Kalman filter is also different. In order to better compare the difference of fractional state model in different order. Figure 1 shows the comparison of fractional state models when the measurement error is very small, that is, only systematic error is considered. The state models of different orders are compared in Figure 1. It can be seen that the difference of state models under different orders is observed. When n_2 is fixed at order 0.9 and n_1 changes from 0.9-0.1, the change of fractional model becomes smaller and smaller, and the higher the order is, the closer the result of fractional state model is to the real value.

5.2 Comparison between Fractional Models and Kalman Filtering

In order to compare the difference between the fractional discrete model and the standard Kalman filter model, the input value u_k is taken as a periodic function.

$$u_k = -0.01(2\pi f_1)^2 \cos(2\pi f_1 t) - 0.015(2\pi f_2)^2 \cos(2\pi f_2 t)$$

where f_1=1.2HZ,f_2=0.3HZ.

When the system noise and the observation noise are fixed, the filtering results of the equations of state of the two models are compared. Table 1 shows the error comparison between Kalman filter model and state model of fractional-order model under different fractional-order derivatives. By changing the order of fractional derivative, the errors of standard filter model and Kalman filter model are compared. In the table below, when the order of fractional derivative is 0.9, 0.8 and 0.7, the error of the fractional model gradually increases, but the error of the fractional state model is always smaller than that of the standard Kalman filter model. It is proved that the established fractional state model can describe the observed data well and has better state estimation ability.

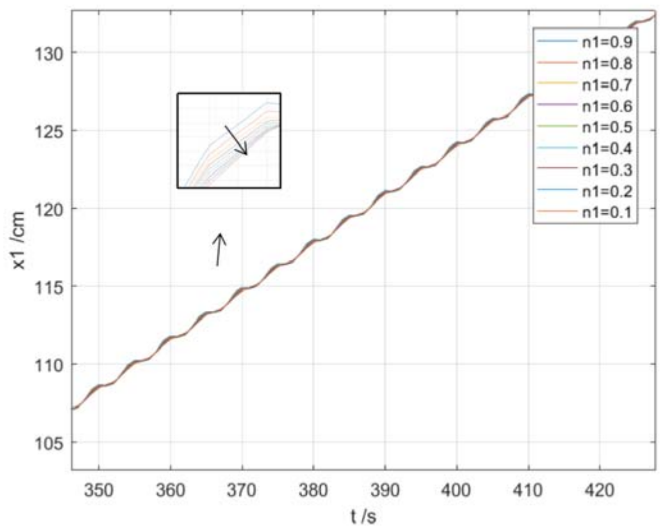

Figure 2: State Model of Fractional Kalman Filter Model with Different Fractional Orders

Table 2 shows the comparison of root mean square error (RMSE) of the results of standard Kalman filtering and fractional filtering in different noise measurement under the same fractional derivative order. The following table shows the RMSE comparison between the fractional model and the standard Kalman filter model when the order is 0.9 and the measurement noise is 0.1, 0.2 and 0.3. When the error variation is small, the RMSE of both the standard Kalman filter and the fractional-order model becomes smaller, but the RMSE of the fractional-order model is smaller than that of the standard filter model, and the fractional-order model has a better effect when the measurement error is larger.

Figure 3 shows the fractional discrete model established. The prediction is the result of the fractional discrete model, actual is the standard discrete model. It can be seen from the figure that compared with the state equation of the standard Kalman filter model, the fractional model can also describe the state model well. By introducing fractional order, the results of the model can converge faster, and the fractional model can get closer to the real value faster. Compared with the standard Kalman filter model, the discrete linear model obtained by the fractional model not only considers the influence of the last observed value on the current value, but also the influence of the observed value at other times on the current value. In this way, not only the established model and error, but also the real value obtained by the equation of state at the previous time should be considered to affect the prediction result of the equation of state. This is why fractional model can converge faster than standard Kalman filter model.

Table 1: Comparison between Kalman Filter State Model and Fractional Model State Model

Position information （cm）	The order of fractional models	Error of Kalman Filter State Model	Errors in Fractional State Models
1	0.9	6.4595	3.4685
1	0.8	6.4595	4.2385
1	0.7	6.4595	4.6072

Table 2: Comparison of filtering RMSE values between Kalman filtering state model and fractional model

Order of fractional models	Measurement error v_k	RMSE of Kalman filtering	RMSE of Fractional Models filtering
0.9	0.3	0.8543	0.6795
0.9	0.2	0.3993	0.3797
0.9	0.1	0.3456	0.2497

According to the periodic function of acceleration, the result of x_2 (velocity) can be calculated, as shown in Figure 4 below, which is the actual and predicted velocity values respectively. Figure 5 shows the calculated velocity error of the fractional filtering model.

Figure 6 (fractional order) and Figure 7 (fractional and standard Kalman filtering) show the displacement errors. By calculating the errors between the two, the following figure is obtained, and the RMSE values before and after filtering are calculated for comparison. According to Figure 6, it can be seen that the fractional filtering model can reduce the interference of noise terms on the results, indicating that the fractional model has good filtering performance. Figure 7 compares the errors of the fractional model and the standard filtering model. It can be seen from the figure that the fractional Kalman filtering model has smaller errors compared to the standard filtering model, and the error fluctuation of the fractional model is smaller, that is, the rate of error change is smaller. This is mainly due to the memory of the fractional model, and the changes of the fractional model are smaller compared to the standard Kalman filtering model.

Figure 8 shows the mean square error results between the optimal estimation and state equation of the fractional model and the standard model simulated at 0.9 orders. After multiple simulations, it can be seen that the fractional model can provide a result with smaller errors compared to the standard model. At the same time, a standard Kalman filtering model was provided for comparison, and the surface fractional model showed better filtering results.

6. Conclusion

In this paper, a fractional Kalman filter model is obtained based on the standard Kalman filter model and applied to the accelerometer model. Through numerical simulation, it is proved that the fractional state model can simulate the accelerometer state well. When the fractional order and measurement error are changed, the filtering results of the fractional Kalman filter and the standard filter model are compared, and the results of RMSE are compared. It is proved that fractional-order Kalman filtering is more stable than integer filtering, and the fractional-order discrete model established has better convergence. In this paper, the results of different orders are compared to verify the reliability of the fractional-order model, and the fractional-order model has a better filtering effect.

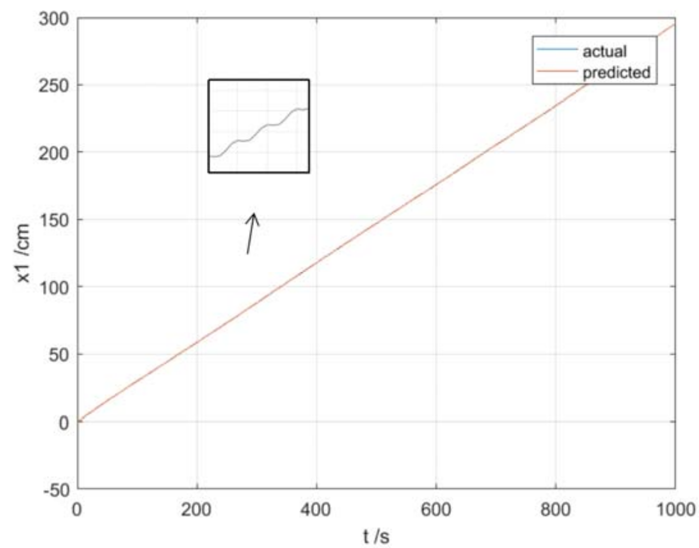

Figure 3: Displacement after fractional model filtering

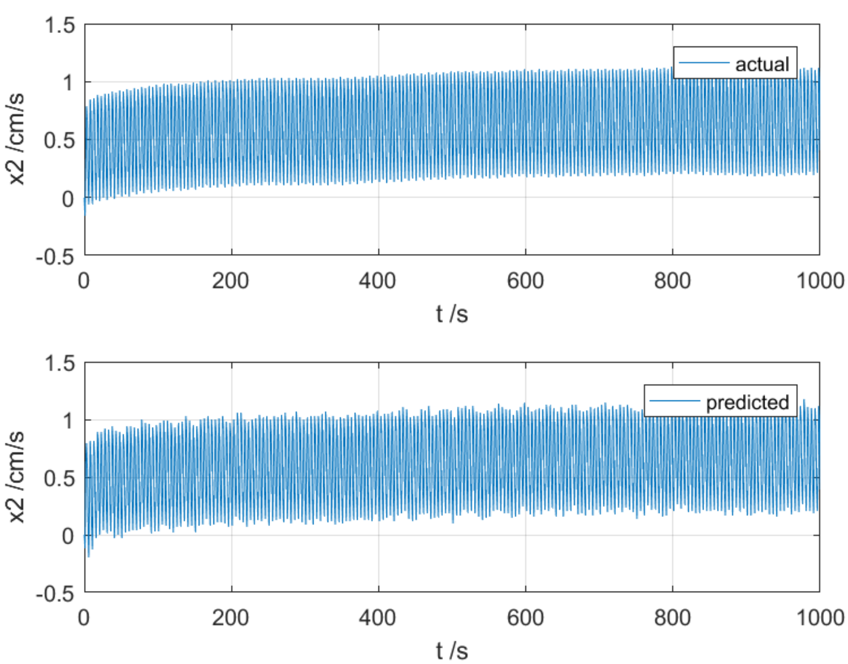

Figure 4: Velocity after fractional model filtering

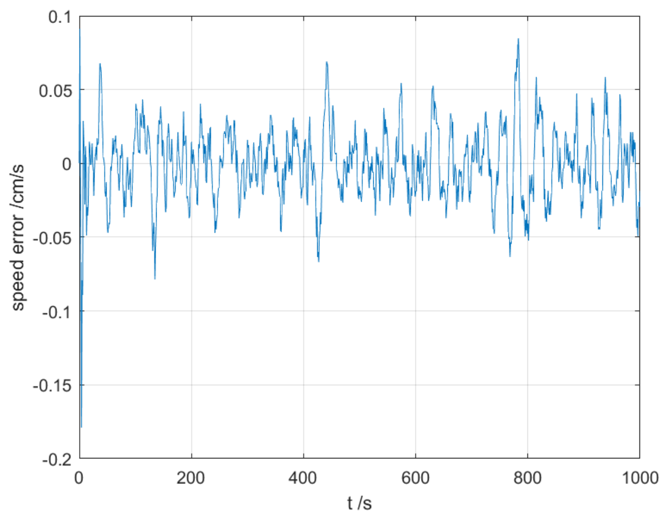

Figure 5: Velocity error after fractional model filtering

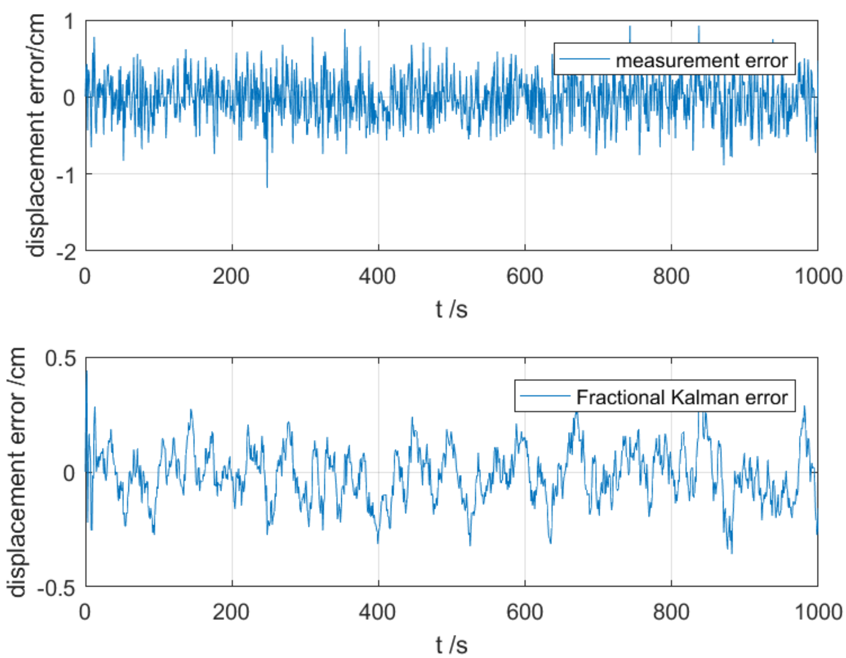

Figure 6: Error after fractional model filtering

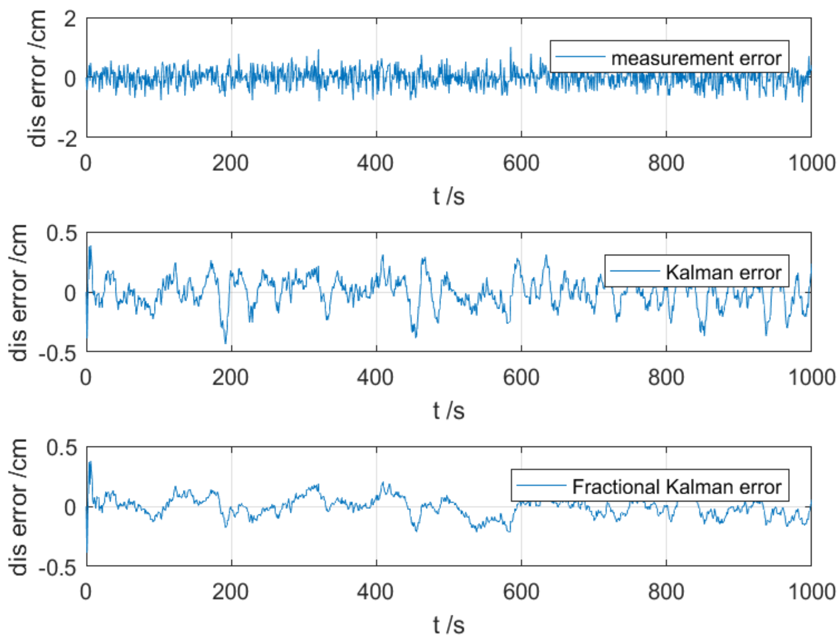

Figure 7: Error after filtering

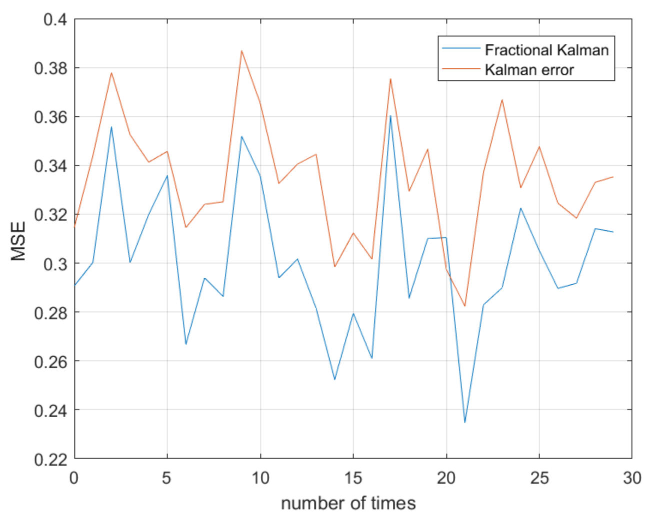

Figure 8: Comparison of mean square error between Kalman filtering model and fractional model

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