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A Survey for GNSS Application of Ionospheric Information Extraction, Modelling, and Forecasting Techniques

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Abstract: Ionospheric information plays a significant role in modern communication and navigation systems. This article provides a comprehensive survey of the application, modeling methods, and results related to ionospheric information. The article first introduces prerequisite knowledge of the ionosphere, and then describes the methods and techniques used in the extraction of ionospheric information, the generation of ionospheric Vertical Total Electron Content (VTEC) maps, the modeling and interpolation of the ionosphere, and the forecasting of ionospheric information. The article also provides illustrative examples and figures to demonstrate the effectiveness of the presented methods. Our survey provides insights and guidance for researchers and practitioners interested in developing ionospheric modeling and forecasting methods for GNSS applications.

Key words: GNSS, ionsophere extraction, nowcasting, forecasting, machine learning.

I Introduction

The ionosphere, occupying the upper echelons of the atmosphere between 60 and 1000 km above the Earth's surface, exists due to the ionization process initiated when high-energy particles, primarily solar radiation, collide with atmospheric particles [1]. The phenomena observed in the ionosphere warrant exploration across diverse fields, including but not limited to aerospace, environmental management, hydrology, soil science, geology, geographic information systems, remote sensing, meteorology, and Earth sciences [2]. This article concentrates on exploring the practical applications and potential contributions of ionospheric research to scientific studies, particularly within the realms of Global Navigation Satellite Systems (GNSS) and BeiDou satellite navigation positioning. Within these contexts, the ionosphere's interference and perturbation effects on radio signals are of central concern. Specifically, as radio signals traverse the ionosphere, the ionospheric free electrons manipulate their propagation characteristics, thus significantly affecting radio systems reliant on these signals. This influence can lead to detrimental impacts on the accuracy of GNSS services and satellite navigation positioning.

Given this backdrop, the imperative to undertake rigorous research on the ionosphere and its associated applications becomes clear: the aim

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is to offset or minimize the ionosphere-induced disruptions to satellite signal propagation and thereby optimize the quality of navigation and positioning services. This objective necessitates a deeper understanding of free electrons and plasma motion, coupled with an extensive comprehension of the overall natural phenomenon. A primary hurdle lies in developing precise models to depict the ionosphere's behavior and predict its effects on radio wave propagation. Conventional models, founded on potentially invalid assumptions, often lead to substantial errors in the forecasted impacts [3]. In contrast, machine learning techniques like deep neural networks [4] can decipher the underlying relationships in the data without rigid assumptions [5], facilitating more accurate predictions. This capacity has substantially boosted our capability to predict ionospheric conditions and develop innovative mitigation strategies. Moreover, the vast quantities of data harvested from multiple sources, including terrestrial and extraterrestrial sensors, have opened new avenues for data-driven ionospheric research. This, in turn, can contribute to remarkable improvements in the performance and reliability of GNSS and BeiDou satellite navigation systems.

Armed with the above technologies, we have formulated effective methods to extract ionospheric information and three distinctive strategies to implement this information, namely, VTEC map construction, ionospheric piercing points information interpolation, and ionospheric information forecasting. The interconnection between them and subsequent services is illustrated in Fig. 1. This article aims to provide a succinct overview of the fundamental principles, methodologies, and corresponding outcomes associated with each approach.

In the ensuing sections of this paper, we will provide a thorough exploration of each of the above-mentioned approaches for ionospheric research. We will focus on their unique attributes, limitations, and future prospects.

Firstly, we will scrutinize the Total Electron Content (TEC) information extraction method, which employs GNSS signals to calculate the TEC of the ionosphere.

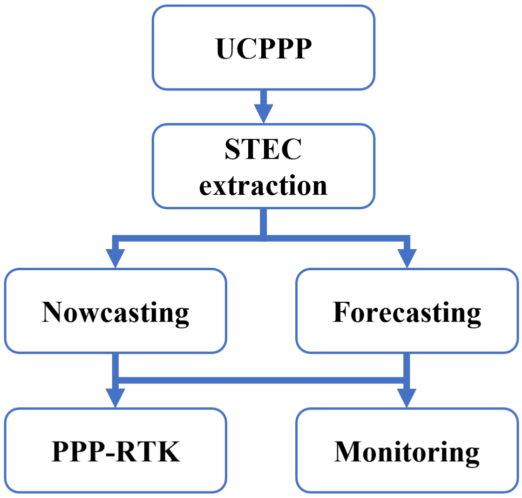


Fig 1: Flow chat of services.

Secondly, we will shift our attention to the construction of VTEC maps. This method leverages a network of GNSS receivers to generate two-dimensional illustrations of the VTEC distribution in the ionosphere.

Next, we will delve into the differential Slant Total Electron Content (STEC) dSTEC interpolation method. This approach is utilized to estimate the temporal and spatial variations of STEC.

Finally, we will investigate the current status of ionospheric information forecasting, examining the models and algorithms used, along with their limitations and challenges.

In conclusion, this paper seeks to provide a comprehensive overview of the key methodologies employed in ionospheric research, spotlighting the significance and potential applications of such research across various scientific and technological fields.

II Ionosphere extraction

Introduction

Two predominant methods are leveraged to derive TEC information within the ionosphere: traditional ionospheric sounding techniques [6] and GNSS-based ionospheric sounding techniques [7]. Traditional ionospheric sounding methodologies encompass two types. The first comprises ground-based radio sounding methods, which include ionosondes, oblique ionosondes, backscatter ionosondes, incoherent scatter radars, and low ionosphere soundings. The second type, known as topside sounding, places a sounding instrument on a low-Earth orbit satellite to glean TEC information between the satellite's altitude and the peak of the ionosphere. Alternatively, GNSS-based techniques for acquiring ionospheric information rely on the ionospheric delay of the GNSS signals. By utilizing the frequency dependency of the ionospheric delay within the positioning equations, one can estimate the TEC information along the GNSS signals' propagation path. This technique has gained widespread acceptance due to its ability to provide real-time, high-precision, and continual ionospheric information. Additionally, the GNSS-based method is highly cost-effective and requires minimal infrastructure, positioning it as a practical solution for both research and operational applications. The extraction of the STEC parameter using the GNSS-based approach also furnishes information on ionospheric irregularities, thereby aiding in the study of space weather phenomena and the understanding of the associated physical processes.

Collectively, the GNSS-based approach serves as an invaluable instrument for ionospheric research and its practical applications. For instance, Ionospheric Pierce Point (IPP) interpolation [8] is utilized to estimate ionospheric conditions at a given location. By interpolating the STEC values procured from proximate GNSS stations, one can estimate the ionospheric conditions at any desired location. This is particularly beneficial for GNSS positioning, which relies on accurate ionospheric corrections to mitigate the effects of ionospheric delay on the GNSS signals. This topic will be discussed in more detail in a subsequent section.

Principles and Methodologies

The GNSS basic observables can be categorized into pseudorange and carrier phase observables.

$$P_{r,f}^{s} = \rho_r^s + c(t_r - t^s) + T_r^s + \gamma_f I_r^s + D_{r,P_f} - D_{P_f}^s + \epsilon_{r,P_f}^s \tag{1}$$

$$\Phi_{r,f}^{s} = \rho_r^s + c(t_r - t^s) + T_r^s - \gamma_f I_r^s + D_{r,\Phi_f} - D_{\Phi_f}^s + \lambda_f N_f + \epsilon_{r,\Phi_f}^s \tag{2}$$

In this article, the pseudorange observables are represented by $P_{r,f}^s$, while the carrier phase observables are represented by $\Phi_{r,f}^s$, where the superscripts s, r, and f denote the satellite, receiver, and frequency band, respectively. ρ_r^s represents the geometric distance between the satellite and receiver, c represents the speed of light, t_r and t^s represents the receiver and satellite clock errors, T_r^s represents the tropospheric delay, I_r^s represents the ionospheric delay, γ_f represents the frequency-dependent ionospheric delay, D_{r,P_f} represents the hardware delay of the receiver of frequency f, $D_{P_f}^s$ represents the hardware delay of the satellite of frequency f, $D_{\Phi_f}^s$ represents the hardware delay of the satellite. λ_f represents the wavelength of frequency f, N_f represents the ambiguity of frequency f, and ϵ_r^s represents other known or unknown errors. The original observation equations of the pseudorange and carrier phase observables can be expressed as follows [9]:

The fundamental observation equation of GNSS can be utilized to formulate the non-differential, non-combination PPP algorithm [9]. When this equation is applied to a reference station with known geometric position and high-accuracy hardware products, it can be combined with precise ephemeris and precise satellite clock products to considerably mitigate the satellite orbit error caused by the model clock and satellite clock bias. Additionally, other related empirical models or products can be used to correct system biases such as atmospheric delay, antenna phase center offset at both satellite and receiver ends, phase ambiguity, relativistic effects, solid tide and ocean tide, and Earth rotation. When amalgamated with appropriate solution algorithms such as least squares and Kalman filtering, the equation can yield the STEC between the satellite and the base station with relative high precision [10, 11, 12, 13].

III Nowcasting

Introduction

The ionosphere nowcasting method is primarily used to study the behavior and characteristics of the ionosphere, and to provide necessary information for forecasting ionospheric conditions. This information is crucial for monitoring the impacts of space weather events on technology-based systems, such as satellite communication and navigation. Space weather events, including solar flares and coronal mass ejections, can cause changes in the ionosphere that can disrupt radio communications and navigation systems, such as GPS. Furthermore, the ionosphere nowcasting method is also employed to help understand the impact of human activities, such as space weather and climate change, on the ionosphere. Accurate ionosphere nowcasting requires the use of advanced data assimilation techniques, which can effectively integrate observational data into models, and accurately capture the complex spatio-temporal variations in ionospheric parameters.

Numerous research studies have been focused on resolving the primary challenge and aiming to achieve precise results, especially for high ionospheric activities. The first category of approaches includes methods that estimate and represent ionospheric corrections as VTEC, such as the Regional Ionospheric Map (RIM). The Wide Area Augmentation System (WAAS) employed the Kriging model to express spatial correlation for regional ionosphere. Observation deviations and information of code noise are then used to adjust the semivariogram [14] [15] . Huang et al. [16] extended the Kriging method, taking into account the accuracy of TEC observations. Liu et al. [17] proposed an adjusted Spherical Harmonics Adding Kriging (SHAKING) method to generate real-time RIMs. The European Geostationary Navigation Overlay Service (EGNOS) used a non-uniform partitioning scheme for the ionospheric grid [18]. However, these methods are affected by modeling and mapping errors and may not be sufficient for high-precision positioning services [19], particularly during high ionospheric activity. The second category of approaches includes methods that estimate and represent ionospheric corrections in the form of STEC and its differential values. Wanninger [20] introduced a location-based linear interpolation model (LIM) to model differential ionospheres in the region, and this model is equivalent to the 2-D low-order surface model [21]. Cui et al. [22] used a distance-based LIM to estimate the values for the user station, and their approach to addressing the above-mentioned difficulty involves selecting observations from the Ionospheric Pierce Points (IPPs) based on certain rules. Xiang et al. [23] divided the value of differential STEC (dSTEC) into a deterministic part and a stochastic part. They then used inverse distance weighting (IDW) and stochastic noise models, such as multipath and modeling error, to estimate the dSTEC of users.

Principles and Methodologies

In this article, we introduce a commonly used and accessible information source: the VTEC map, along with its construction approach and modeling methods. As mentioned previously, the ionosphere is a high-altitude layer located between 60-1000 km above the Earth's surface, with a certain thickness. In order to simplify the description of the variations in Total Electron Content (TEC) of the ionosphere, it is commonly assumed that all free electrons in the ionosphere are concentrated on a perfectly thin spherical shell at a height of 350-450 km, which is referred to as the Single Layer Model (SLM) of the ionosphere.

Using the aforementioned GNSS observation equation, the electron content on the signal propagation path, i.e., STEC, can be calculated. To obtain the vertical electron content, VTEC, see Figure. 2 ,a projection transformation is required, where $F(z)$ represents the projection function of the ionosphere, R is the location of the receiver, and intersects with the thin layer at the piercing point P' along the line of sight to the satellite. The free electrons along the OP' direction are concentrated at point P. The relationship between the projection function and the STEC and VTEC is given by [24]:

$$F(z) = (1 - (\frac{R_E}{R_E + h} \sin z)^2)^{-\frac{1}{2}} \tag{3}$$

where, z is the zenith angle to satellite, h is the height of ionosphere layer presumed, R_E is the average radius of Earth.

Once obtaining the VTEC map information,

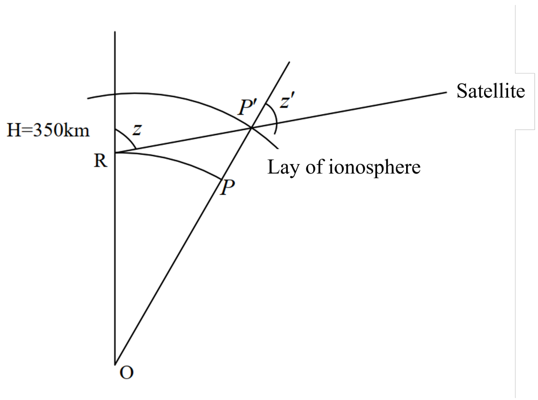


Fig 2: Diagram for VTEC mapping caculation

the model can be established to nowcast information for unknown positions. A classical model for ionospheric VTEC spatial information we will introduce as one paradigm of modeling method, which can be utilized for interpolation, named as spherical harmonic function model. The spherical harmonics are a set of orthogonal functions defined on the surface of a sphere. They can be thought of as the natural extension of the Fourier series to the surface of a sphere. To model a complex function as ionosphere VTEC information using spherical harmonics [25], the function is first decomposed into a linear combination of spherical harmonics. This is done by projecting the function onto each of the spherical harmonics and calculating the coefficients of the expansion. Once the expansion coefficients are obtained, the original function can be reconstructed by summing the individual spherical harmonics, weighted by their respective coefficients.

$$
\begin{aligned}
VTEC(\phi, \lambda) &= \sum_{n=0}^{n_{max}} \sum_{m=0}^{n} \hat{P}_{nm}(\sin\phi)(A_{nm}\cos(m\lambda) \\
&\qquad + B_{nm}\sin(m\lambda)) \\
\hat{P}_{nm}(\sin\phi) &= N(n,m)P_{nm}(\sin\phi) \\
N(n,m) &= \sqrt{\frac{(n-m)!(2n+1)(2-\delta_{0m})}{(n+m)!}}
\end{aligned}
\tag{4}
$$

where $VTEC(\phi, \lambda)$ represents VTEC of ionospheric piercing point at latitude ϕ and longitude λ. n_{max} represents maximum degree of spherical harmonic function. $P_{nm}(\sin\phi)$ represents n degree and m order Unnormalized associated Legendre function. δ_{0m} represents Kronecker theta function. A_nm and B_nm are model variables to be estimated. Once we obtain a spherical harmonic function model from observational data, we can simply obtain the VTEC value at a given latitude and longitude coordinate by interpolation.

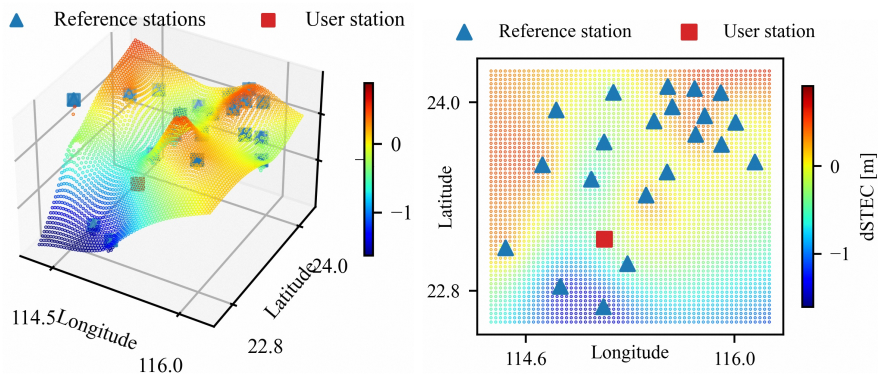
Another classical methodology for modeling and interpolation for VTEC spatial information is surface function fitting, which refers to the process of fitting a continuous surface function to a set of discrete data points. A particular example [26] for surface function fitting on 2-dimensional sptial space can be formalised as solving the following mathematical programming:

$$
\begin{aligned}
&\arg\min_{f \in W^{2,2}} \sum_{i=1}^{N} (s_i - f(p_i))^2 + \lambda J[f] \\
&J[f] = \int\int_{\mathbb{R}^2} \Big[\Big(\frac{\partial^2 f(x)}{\partial p_1^2}\Big)^2 + 2\Big(\frac{\partial^2 f(x)}{\partial p_1 \partial p_2}\Big)^2 + \\
&\qquad \Big(\frac{\partial^2 f(x)}{\partial p_2^2}\Big)^2\Big] \,\mathrm{d}p_1 \,\mathrm{d}p_2
\end{aligned}
\tag{5}
$$

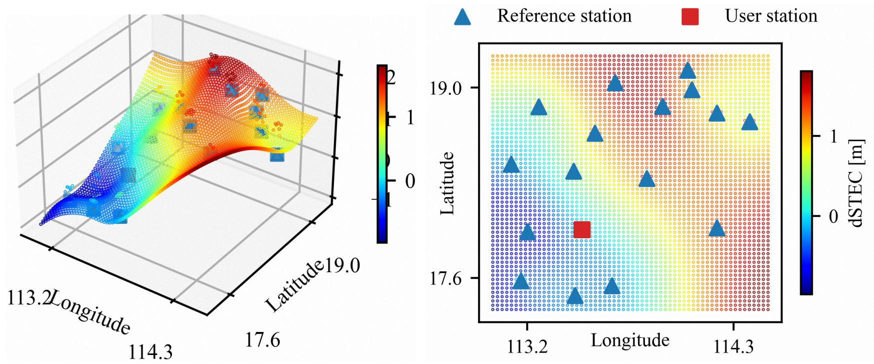
where f represents the model function under consideration, whose form is implicitly determined by the variations problem. The solution of the above programming actually provide a function that given any coordinates it generate corresponding VTEC values.

Results Illustration

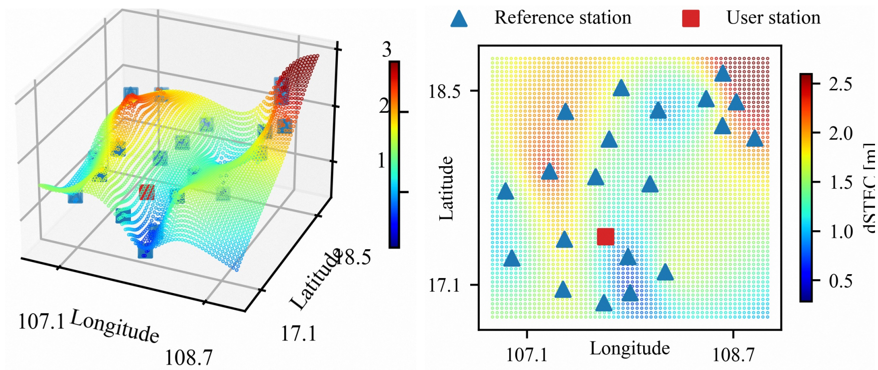
In this section, we present the nowcasting information constructed in our study. The Figure. 5 displays the VTEC map for the entire China region, and subsequent Figure. 4 in the series depict the VTEC information for specific regions. As seen, the VTEC map generated through the nowcasting technique has proven to be a highly effective means of capturing ionospheric information across a large region, even in practical scenarios, where only a limited number of satellite observations are available for calculating STEC information, compared to vast territory. Furthermore, the ionospheric information modeling technique enables us to provide highly accurate STEC corrections by delicately constructing region representative functions that potentially characterize the spatial correlation between known reference stations. The resulting ionospheric maps, which are illustrated in the Figure. 3, provide a vivid depiction of the effectiveness of this approach.



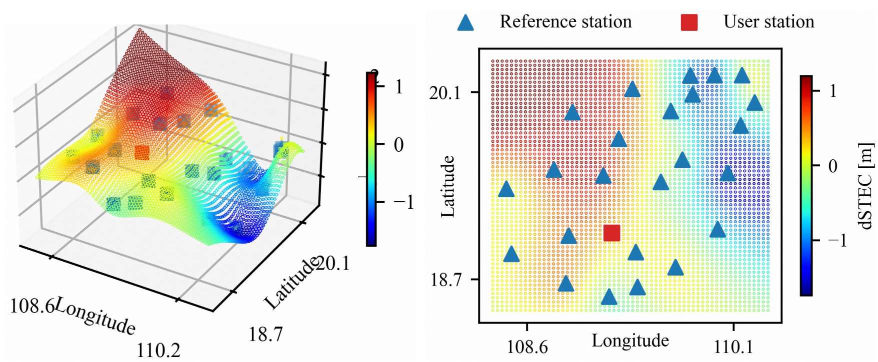
(a) Example 1



(b) Example 2



(c) Example 3



(d) Example 4

Fig 3: The four examples represent the results at four different time points. The left figures represent longitude and latitude on the x and y axes respectively, with the third dimension representing STEC values. The right figures are a top-down view of the left figures. Each of them represents a sptial-temporal IPP TECU modeled values. Observed that the distribution of ionospheric values is complex and irregulary. Unit is meter.

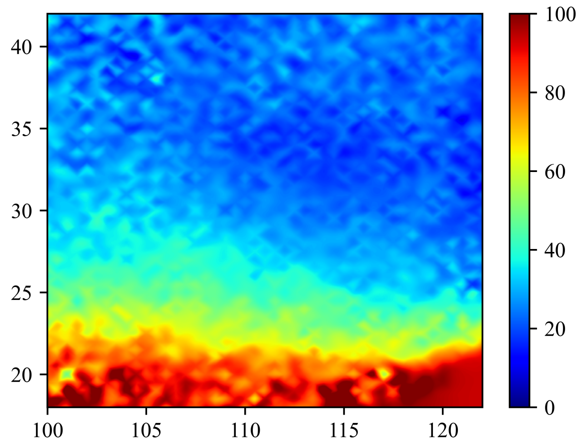


Fig 4: Nowcasting of a specific sptial region. The horizontal and vertical axes in the figure represent longitude and latitude respectively. The bar located on the right side of the figure serves to illustrate the extent of TEC. Unit is TecU.

IV Forecasting

Introduction

The prediction of the ionospheric state is a complex problem that has been the subject of extensive research in the field of space science and technology. In this study, we have chosen to focus specifically on the forecasting of vertical total electron content (VTEC) maps, given that VTEC is widely recognized as an effective indicator of the ionosphere's effects on satellite-based navigation and communication systems.

There are a variety of methods that have been proposed to tackle the prediction of ionospheric conditions, which can broadly be divided into two categories: physical methods and data-driven methods. Physical methods, such as the SAMI2 and SAMI3 models [27], [28], rely on the representation of plasma behavior along the earth's dipole field through the application of physics-based equations. While these methods can provide insightful predictions, they are often computationally demanding and require a large and diverse set of data inputs, which limits their practical usefulness for real-time applications.

In contrast, data-driven methods take a more empirical approach, relying on mathematical models that are trained on observational data. These methods are more computationally efficient and less data-intensive than physical methods, and they can provide a useful characterization of the ionosphere's patterns, even in cases where detailed physical data is not available. In recent years, a number of data-driven methods have been developed that have demonstrated promising results for both modeling and forecasting of ionospheric conditions. For example, Liu et al. [29] introduced a harmonic cap model to forecast regional VTEC, based on dual-frequency observations from the Global Positioning System (GPS). Wang et al. [30] proposed an adaptive autoregressive model for predicting global VTEC maps, while Erdogan et al. [31] combined B-spline functions with Kalman filtering to simulate the spatial and temporal dynamics of the ionosphere on a global scale. Additionally, Liu et al.[17] used neural networks to predict spherical harmonic function parameters for representing ionospheric information.

While these data-driven methods are capable of capturing some of the important features of VTEC data, they are nonetheless limited by the mathematical functions used to model the data, which may not fully capture the complex temporal and spatial dynamics of VTEC. Nevertheless, these methods represent important advances in the field of ionospheric prediction, and will continue to play a critical role in the ongoing development of GNSS services and applications.

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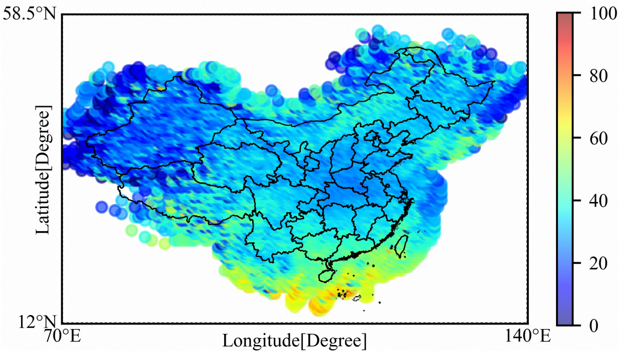
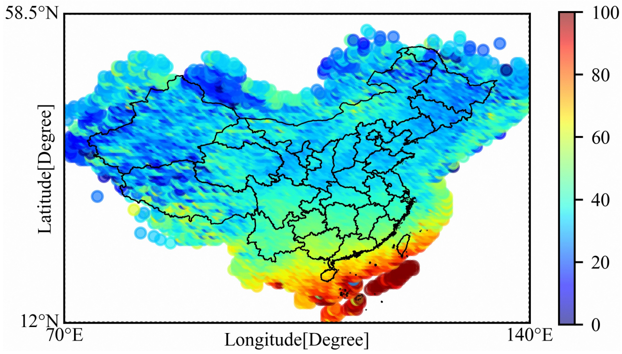
Principls and Methodologies

In general, a comprehensive and mighty forecasting model should be capable of predicting ionospheric information for any given spatial coordinate and any moment that is not too distant from the present. This requires the model to effectively characterize the spatial-temporal features in a unified manner. Such a model should exceed the capabilities of models that estimate temporal and spatial features separately and then combine them.

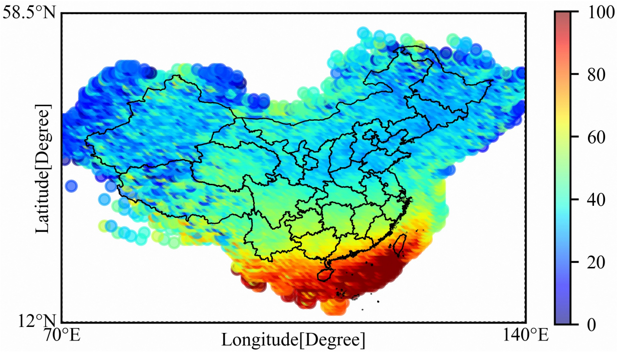
However, in this article, we focus on a type of less holistic models that only forecast ionospheric information (VTEC, for example) at given grids. A general framework for such kind of forecasting



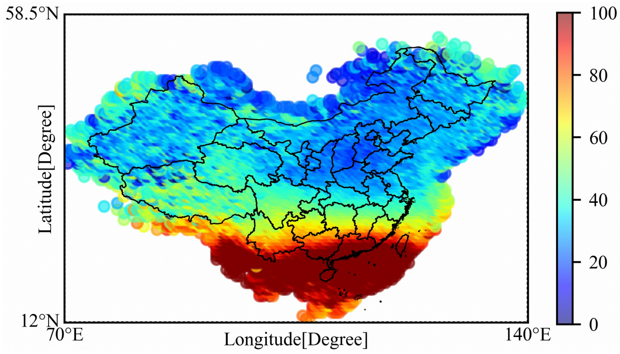
(a) Low activity period



(b) Middle activity period

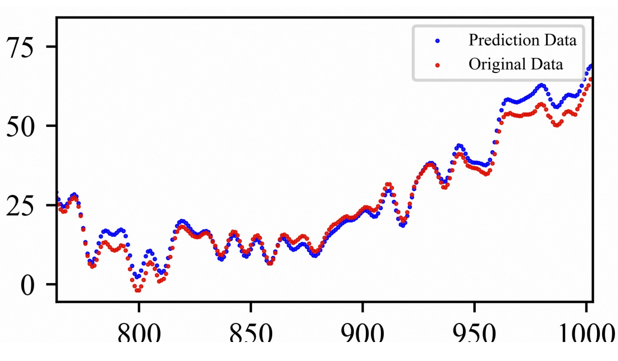


(c) High activity period

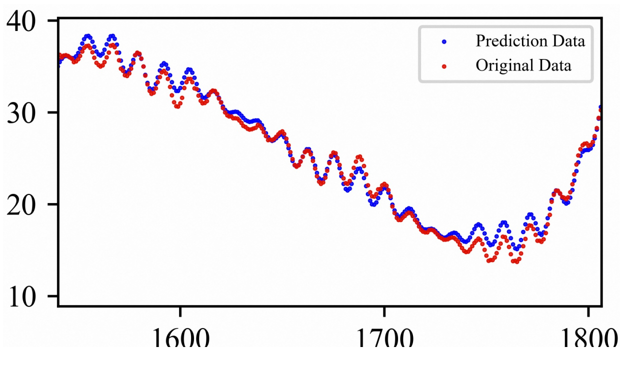


(d) Severe activity period

Fig 5: China VTEC maps. The horizontal and vertical axes in the figure represent longitude and latitude respectively. The bar located on the right side of the figure serves to illustrate the extent of TEC. Unit is TecU.



(a) Forecasting result 1



(b) Forecasting result 2

Fig 6: Two figures represent the different time period. The red line and the blue line represent the predicted and true Vtec values. Forecasting VTEC at given grid for the next three hours.The vertical axis in the image represents Vtec values, with a unit is TecU, and the horizontal axis represents the duration of a certain time period starting from a specific moment, with a unit of 5 minutes. Observed that the fluctuations are quite severe.

model can be formally stated as follows:

Given $M \times N$ grid points, each assigned with a value that represents the VTEC at that point, obtained through the mapping of STEC to the zenith direction as described previously, the value of VTEC at each point varies over time. The observations of the grid at any given time t can be represented by a matrix $\mathcal{X}_t \in \mathbb{R}^{M\times N}$. If periodic observations are recorded, a sequence of matrices $\mathcal{X}_1, \mathcal{X}_2, ..., \mathcal{X}_t$ can be obtained. With J historical observations, the current observation, and a defined time period K at hand, the central objective lies in forecasting the optimal sequence of matrices for the forthcoming instances. [32]:

$$\overline{\mathcal{X}}_{t+1}, \overline{\mathcal{X}}_{t+2}, \cdots, \overline{\mathcal{X}}_{t+K} = \underset{\mathcal{X}_{t+1},\mathcal{X}_{t+2},\cdots,\mathcal{X}_{t+K}}{\arg\max} p(\mathcal{X}_{t+1}, \mathcal{X}_{t+2}, \cdots, \mathcal{X}_{t+K} | \mathcal{X}_{t-J+1}, \mathcal{X}_{t-J+2}, \cdots, \mathcal{X}_t) \tag{6}$$

A intuitive idea for constructing forecasting model in this setting is to design a tansfer function that convolves the information from a given point and the points around it, and maps it to the information at the same point in the next moment. With the aid of machine learning techniques, we can train the model to optimize the function parameters and minimize predetermined performance metrics.

As previously mentioned, there are at least two ways to model such a transfer function: by deriving it from physical insights, such as physical equation modeling, or by utilizing a data-driven approach, such as neural networks. A commonly employed data-driven method for ionospheric information forecasting involves a set of procedures. Firstly, a parameterized spatial regression function is employed to establish the spatial model for each moment. Secondly, a time series forecasting method is applied to provide the next set of parameters. Thirdly, these forecasted parameters are integrated into the spatial model to generate the final results. It is worth noting that this method requires a substantial amount of data to accurately establish the spatial and temporal relationships between the various parameters. Moreover, the accuracy of the final results heavily depends on the quality and quantity of the input data.

Results Illustration

In this section, we present the forecasting results of VTEC values for two selected grids over a three-hour time period. As observed in Figure. 6, the fluctuations during the testing period are quite severe, and a robust forecaster should be able to accurately track these variations.

V Conclusion

In conclusion, this article has provided a comprehensive overview of the knowledge, methods and results related to the ionosphere, including ionosphere information extraction, VTEC map generation, ionospheric modeling and interpolation, and ionospheric information prediction. The ionosphere is a crucial layer in the Earth's atmosphere, which plays a significant role in a range of fields such as aviation, space technology, natural resources and environment, hydrology, soil, geology, geographic information systems, remote sensing, meteorology, and Earth sciences. In particular, the study of the ionosphere's dynamic changes and accurate prediction of its future variations are essential for ensuring the smooth operation of wireless communication systems, improving the accuracy of satellite navigation systems, and promoting the theoretical and practical applications of ionospheric physics in the field of Earth sciences. The methods and results presented in this article can serve as a valuable resource for researchers and practitioners in the field of ionospheric research, enabling them to effectively extract and analyze ionospheric information, generate accurate VTEC maps, model and interpolate ionospheric data, and predict the dynamic changes of the ionosphere in real-time. Overall, this article highlights the importance of ionospheric research and its impact on a range of scientific and technological applications, emphasizing the need for further research and development in this field to address the challenges and opportunities posed by the dynamic and complex nature of the ionosphere.

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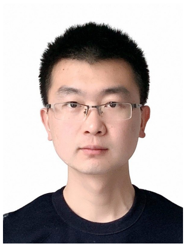

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