

Two-stage fusion localization based on UWB/PDR/Geomagnetism in underground space

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Abstract: A two-stage fusion positioning model based on UWB/PDR/Geomagnetism is proposed for pedestrian positioning in underground space. Firstly, an improved particle filter (PF) based on regional constraint is applied to PDR/Geomagnetism combination positioning, where the PDR positioning results are used to constrain the geomagnetic matching region. The proposed algorithm improves the inherent blindness of scattered particles of traditional PF, thus to enhance the positioning accuracy. Furtherly, Factor graph (FG) is used to fuse the output of PF above with the UWB positioning results, which effectively overcomes the serious problem of positioning hopping points caused by signal occlusion in some areas of UWB system. Experimental results show that the improved PF can outperform extended Kalman filter (EKF) for PDR/Geomagnetism combination positioning and FG algorithm can provide a higher positioning accuracy for UWB/PDR/Geomagnetism fusion positioning.

Keywords: Underground space; Pedestrian Dead Reckoning; Geomagnetic matching positioning; Ultra WideBand; Particle filter; Factor graph

I. Introduction

With the rapid development of society and technology, utilization of underground space is

getting more and more extensive, from shallow utilization to large-scale development, from solving urban problems to enhancing urban competitiveness, the intensive and composite utilization of underground space resources has been regarded as a standard paradigm to support the sustainable development of urban modernization. The challenge of pedestrian localization in underground space is mainly reflected in its closed characteristics, which is a typical radio denial environment. And the existing GNSS, WiFi, 4G and other wireless networks cannot be valid. Due to the increasing demand for location-based services, research on effective pedestrian localization in underground space becomes a hot topic due to the increasing demand for Location Based Service (LBS)^[1-2].

At present, indoor positioning technology for underground space has been in-depth research by scholars, and indoor positioning technology based on geomagnetism, visible light, inertial guidance and UWB has been widely used and developed^[3-6]. Among many technologies, UWB has recently become effective solution to high positioning accuracy and strong anti-interference. However, it also has its own shortcomings, which is the serious attenuation of positioning accuracy in areas where the signal is occluded. Based on this, it is considered to integrate PDR and geomagnetism, two positioning

technologies that do not require additional infrastructure.

Many studies have been carried out on PDR and geomagnetism, as well as UWB and PDR combined localization methods. As reported briefly in [7], an IMU localization algorithm based on magnetic constraints uses multi-stage Fourier transform method to extract features, whose average error of the trajectory under loop closure constraint is controlled below 2.15 m. Song Biao et al.^[8] designed an inertial guidance-assisted geomagnetic indoor positioning system for mobile phones, which achieved good positioning accuracy in two-dimensions, but could not adapt well to three-dimensional complex environment. Reference [9] aiming at the shortcomings of low recognition of geomagnetic signals and the problem of error accumulation of PDR, the particle filter algorithm is used to fuse the geomagnetism and PDR positioning results, whose positioning error can reach more than 80% within 1.5m. An improved particle filtering method is proposed in [10]. Comparing with the traditional particle filter, multi-dimensional dynamic time warping (MD-DTW) is used to constrain the length of the particle sequence and the method of segmented particle weighting, which can effectively accelerate the convergence speed of particle filtering. As reported briefly in [11], the methods of dead-assisted UWB positioning and UWB-assisted dead reckoning are adopted, which enhances the advantages of UWB positioning, and the maximal positioning error is within 2m. Zhu Caijie^[12] proposed an algorithm using INS to assist UWB positioning, which can effectively make up for the shortcomings of UWB positioning technology in the positioning process. And in terms of the mean positioning error, the algorithm based on Kalman filter (KF) can improve 0.1m in x-axis and 0.2m in y-axis. Benzerrouk put forwards to use cubature Kalman filter (CKF) as a superior alternative to standard filters which improves the mean and covariance propagation consequently^[13]. However, the above methods mainly depend on filter fusion methods, and when the information of one sensor fails or becomes unavailable, Kalman filter and other complex system

are required to be reconstructed to deal with the problems, resulting in disadvantages such as high cost-effectiveness ratio and large memory consumption. For wireless communication positioning methods, the visibility of the signal is one of the factors that affect the positioning accuracy. Under the influence of non-line-of-sight (NLOS)^[14], there are some hopping points in all positioning reference points. Reference [15] describes one base station (BS) based distance and angle positioning algorithm with extended Kalman filter (DAPA-EKF) in NLOS environment, which significantly outperforms the reference methods under various NLOS situations. Mingxiang Liao et al.^[16] presents a Chan-Taylor-Kalman (CTK) joint positioning algorithm based on UWB and adapted to NLOS environment, which indicates that the probability of the positioning error of the CTK joint positioning algorithm greater than 10cm is only about 10%.

To make full use of various indoor positioning methods, some scholars propose to construct a data fusion algorithm framework with dynamic topology based on the relevant theory of FG^[17-18]. The fusion algorithm framework based on the factor graph model can effectively solve the problem of a sensor failure in data fusion, and has good scalability for multiple sensors. Therefore, it can flexibly configure sensors^[19-20]. Factor graph is a kind of probability graph model, first used in the field of coding^[21-22], has gradually been applied to artificial intelligence, positioning navigation and other fields in recent years. One of the applications in the field of navigation is information fusion positioning system. Moreover, factor graph can realize the plug and play of system sensors and has been widely concerned in the field of combined navigation.

In this paper, a two-stage fusion localization model based on UWB/PDR/geomagnetism is proposed. This paper is organized as follows. In section II, it introduces the concret detail about two-stage fusion localization model. Improved PF combination positioning method based on PDR/geomagnetism and FG fusion positioning method base on UWB/PDR/geomagnetism are included. Section III conducts numerical

experiments to evaluate positioning performance of the proposed algorithm. And conclusions are given in last section.

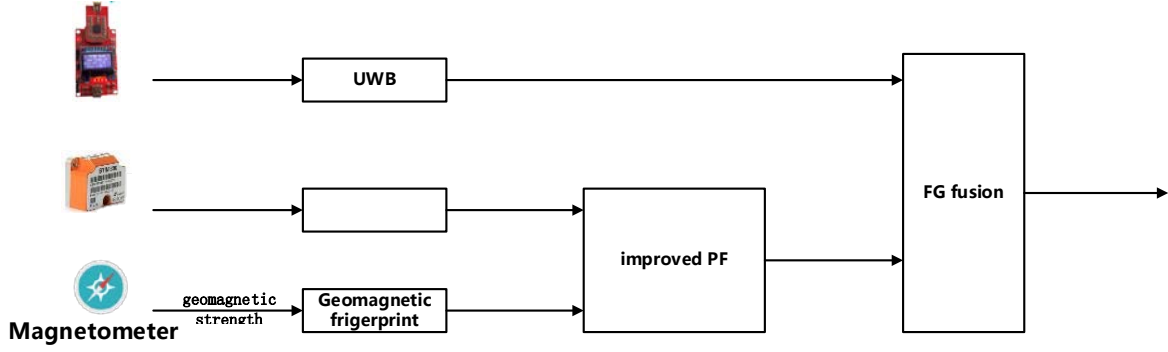


Fig.1 Two-stage fusion localization model

Two-stage fusion localization model is shown in Fig.1. The improved PF algorithm using regional constraints combines the PDR and geomagnetism localization. FG is employed for UWB/PDR/geomagnetism multi-source fusion positioning, which effectively improves the continuity and stability of pedestrian positioning in underground space.

Improved PF Fusion Algorithm for PDR/Geomagnetism Combination

In the first stage of localization, the positioning error of PDR accumulates seriously over time. Geomagnetism positioning employs fingerprint matching method, which has big positioning error because of low fingerprint resolution within a large area. In order to decrease the mismatching rate, the improved PF is used for PDR/geomagnetism combination. PDR positioning result is employed to constrain the fingerprint database area while geomagnetism positioning result corrects PDR positioning result. The PF employs a group of particles to approximate a posterior probability distribution effectively estimating nonlinear non-Gaussian systems. Let the equations of state and observations for nonlinear systems be:

$$\mathbf{x}_k = f_k(\mathbf{x}_{k-1}, \boldsymbol{\omega}_{k-1}) \quad (1)$$

$$\mathbf{z}_k = h_k(\mathbf{x}_k, \mathbf{v}_k) \quad (2)$$

II. Two-Stage Fusion Localization Model and Algorithm

In (1) and (2), \mathbf{X}_k is the system states, $\boldsymbol{\omega}_k$ is the process noise and \mathbf{V}_k is the observed noise.

The state prediction equation can be expressed as:

$$p(\mathbf{x}_k | \mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k | \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} | \mathbf{z}_{1:k-1}) d\mathbf{x}_{k-1} \quad (3)$$

The state update equation is:

$$p(\mathbf{x}_k | \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{z}_{1:k-1})}{\int p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{z}_{1:k-1}) d\mathbf{x}_k} \quad (4)$$

The essence of PF is to use the sum of finite sample points instead of integral operations, that is, the posterior probability density distribution of the target state at k is shown in (5).

$$p(\mathbf{x}_{0:k} | \mathbf{z}_{1:k}) \approx \sum_{i=1}^N w_k^i \delta(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^i) \quad (5)$$

Assuming that $\{\mathbf{x}_{0:k}^i\}_{i=1}^{N_s}$ is the set of particles, N_s is the total number of particles, which can be obtained from the important density function $q(\mathbf{x}_{0:k} | \mathbf{z}_{1:k})$, then the particle weights can be expressed as:

$$w_k^i \propto \frac{p(\mathbf{x}_{0:k}^i | \mathbf{z}_{1:k})}{q(\mathbf{x}_{0:k}^i | \mathbf{z}_{1:k})} \quad (6)$$

In (6), w_k^i is the weight of the i -th particle at k , and $\mathbf{x}_{0:k}$ is the set of states of the system from 0 to k . Sequential importance sampling is the basis for PF implementation, so Samples are drawn from the importance density function which is set to an

easily achievable prior probability. Resampling is to solve the degradation phenomenon in the process of particle transfer, which refers to the imbalance of weight distribution due to the increasing variance of weight. When the filter is seriously degraded, a large number of computing resources are wasted on particles with small weights, which affects the performance of the filter. During the resampling process, particles with large weights produce more particles while particles with small weights produce fewer particles or even discard them directly. However, this will lose the diversity of particles and affect the performance of particle filtering.

In this paper, improved PF based on regional constrain for PDR/geomagnetism is proposed. The PDR positioning result and geomagnetic strength of fingerprint database are taken as the state quantity and the real-time geomagnetic strength is taken as the observation. In the process of initializing the particles, the particles are scattered in the circle determined by the PDR positioning result as the center of the circle and the mean positioning error empirical value as the radius. A schematic diagram of regional constrain positioning is shown in Fig.2.

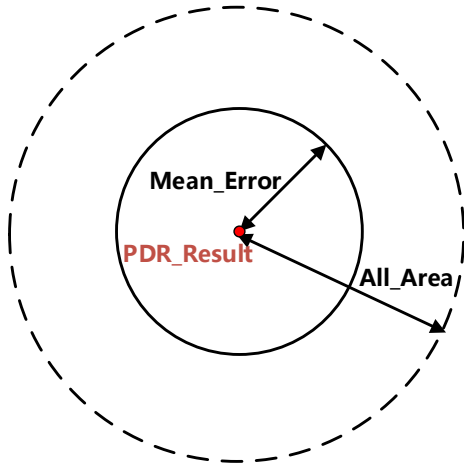


Fig.2 constrained area diagram

In the figure, the dotted circular area is the geomagnetic matching full area, and the solid circular area is the constrained geomagnetic matching area.

The improved particle filter algorithm simplifies the prediction step in traditional PF, improves the inherent blindness of particle seeding, and reduces the number of matching points with the geomagnetic fingerprint database, thus to increase the algorithm

efficiency and decrease mismatching rate.

FG Fusion Algorithm for UWB/PDR/Geomagnetic Fusion

In the second stage of localization, UWB positioning has some hopping points because of NLOS environment. FG is employed for UWB/PDR/geomagnetism fusion positioning. FG is a two-way graph based on factoring a global function with multiple variables to obtain the product of several local functions. Its basic structure, shown in Fig.3, consists mainly of variable nodes (represented by diamonds) and function nodes (represented by circles).

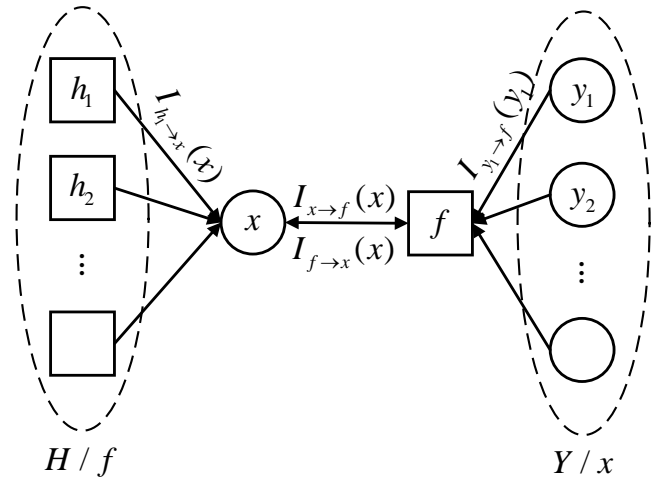


Fig.3 the sum-product algorithm

Sum product algorithm (SPA) is commonly employed to delivery message in factor graphs. Obeying the following guidelines, variable nodes to function nodes messaging:

$$\mu_{x \rightarrow f}(x) = \prod_{H/f} I_{h \rightarrow x}(x) \quad (7)$$

Messaging from function nodes to variable nodes:

$$\mu_{f \rightarrow x}(x) = \sum_{\sim x} \{f(Y) \prod_{Y/x} \mu_{y \rightarrow f}(y)\} \quad (8)$$

Where H represents the set of all function nodes connected to the variable node x union; similarly, Y represents neighbors connected to the f set of function nodes; H/f represents a neighborhood set connected to variable node x , but not included in function node f ; similarly, Y/x

means that it is connected to the function node f neighborhood set, not the variable node x .

For the problem to be solved in the paper, we construct a model as follows. At time k , the state of the location target is expressed as

$$F(k) = (x_k, y_k, z_k) \quad (9)$$

where x_k, y_k, z_k indicates positioning result.

Taking discrete time model, we can get the following results.

$$x_k = x_{k-1} + w_{x_k} \quad (10)$$

$$y_k = y_{k-1} + w_{y_k} \quad (11)$$

$$z_k = z_{k-1} + w_{z_k} \quad (12)$$

where $w_{x_k}, w_{y_k}, w_{z_k}$ stand for measurement noise of position.

The initial state of location target is defined as

$$F(0) = (x_0, y_0, z_0) \quad (13)$$

According to the problem formulation, A factor graph structure model is designed as shown in Fig.4. We select different kinds of location sources which includes PDR/geomagnetic and UWB single-mode positioning result to participate in fusion process. F_{k-1} and F_k respectively represent the state of positioning target in time $k-1$ and k .

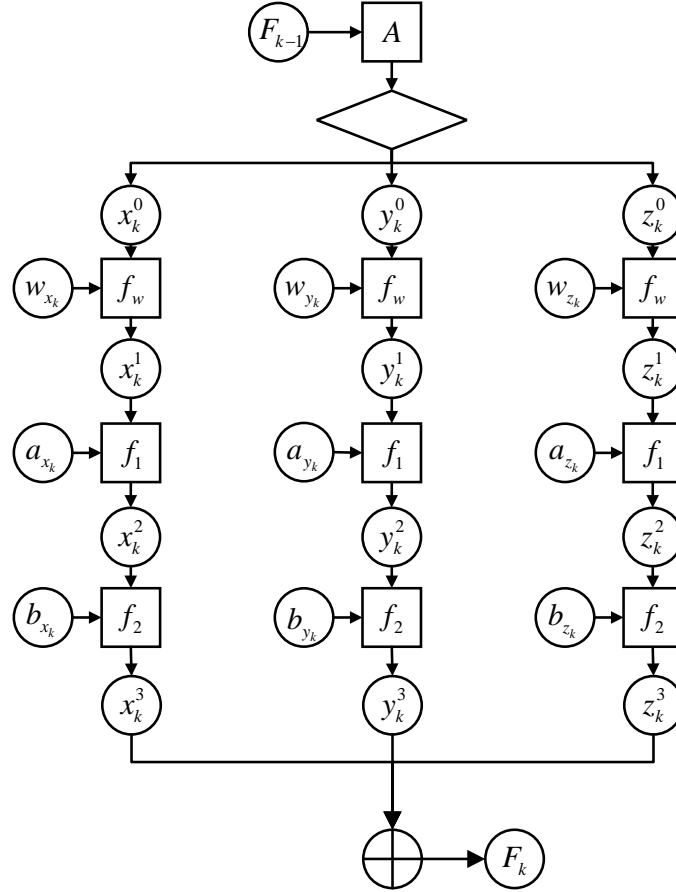


Fig.4 factor graph structure model diagram.

Since the model adopts three-dimensional coordinates, we divide the motion state into three parts: x, y, z , taking the x direction as an example, $x_k^1, x_k^2, \dots, x_k^n$ is the x -coordinate value of different fusion steps at k time, and w_{x_k} is the

measurement noise. a_{x_k}, b_{x_k} are PDR/geomagnetic and UWB single-mode positioning result. f_w represents a fusion function node. Fuse different variable nodes in function nodes. After a series of

soft information calculations, the fusion result x_k^n is obtained. y_k^n and z_k^n can be implemented in the same way. The fusion results x_k^n , y_k^n , and z_k^n were combined to obtain F_k .

During the fusion process, assuming that the soft message passed in the factor graph obeys Gaussian distribution with mean being m and variance being σ . We define the soft-information as

$$I = \{m_I, \sigma_I\} \quad (14)$$

Noting that the product of any Gaussian Probability Distribution Functions (PDF) is still a Gaussian PDF, so we have the following formula

$$\prod_{i=1}^n N(x, m_i, \sigma_i^2) \propto N(x, m_\Omega, \sigma_\Omega^2) \quad (15)$$

where

$$I_{x_k^n} = \left\{ \frac{1}{\frac{1}{\sigma_{x_k^{n-1}}^2} + \frac{1}{\sigma_{a_{x_k^{n-1}}}^2} + \dots} \left(\frac{m_{x_k^{n-1}}}{\sigma_{x_k^{n-1}}^2} + \frac{m_{a_{x_k^{n-1}}}}{\sigma_{a_{x_k^{n-1}}}^2} + \dots \right), \frac{1}{\frac{1}{\sigma_{x_k^{n-1}}^2} + \frac{1}{\sigma_{a_{x_k^{n-1}}}^2} + \dots} \right\} \quad (19)$$

wherein $m_{x_k^n}$ is the final fusion result in x-coordinate shown as m_x .

$$m_x = \frac{1}{\frac{1}{\sigma_{x_k^{n-1}}^2} + \frac{1}{\sigma_{a_{x_k^{n-1}}}^2} + \dots} \left(\frac{m_{x_k^{n-1}}}{\sigma_{x_k^{n-1}}^2} + \frac{m_{a_{x_k^{n-1}}}}{\sigma_{a_{x_k^{n-1}}}^2} + \dots \right) \quad (20)$$

Consistently, m_y and m_z can be obtained in the same way. At this point, we get the state of the k moment. In the same way, we can get the state at $k+1$, $k+2$, and so on until we get the final fusion

$$\frac{1}{\sigma_\Omega^2} = \sum_{i=1}^N \frac{1}{\sigma_i^2} \quad (16)$$

$$m_\Omega = \sigma_\Omega^2 \sum_{i=1}^n \frac{m_i}{\sigma_i^2} \quad (17)$$

Here is some initialization information. the initial value of soft-information is $I_{x_k^0} = \{m_{x_k^0}, \sigma_{x_k^0}^2\}$ with initial mean x_0 and initial variance σ_0^2 . $I_{w_k} = \{m_{w_k}, \sigma_{x_k^0}^2\}$ indicates the indeterminacy of the model.

In Fig.3, according to the sum-product algorithm, we can achieve the soft-information as followed

$$I_{x_k^1} = \{m_{x_k^0} + m_{w_{x_k}}, \sigma_{x_k^0}^2 + \sigma_{w_{x_k}}^2\} \quad (18)$$

Through a series of soft message solution, the n-th variable is obtained. The soft message at the node is

result.

III. Experimental Result

In order to verify the effectiveness of the proposed algorithm, experiments are conducted in actual underground space environment using real data. The data acquisition equipment is one-tag integrated UWB DWM1000 module and IMU module and four-base stations also containing DWM1000 module. The block photograph of BS signal is as follow in the Fig.5.

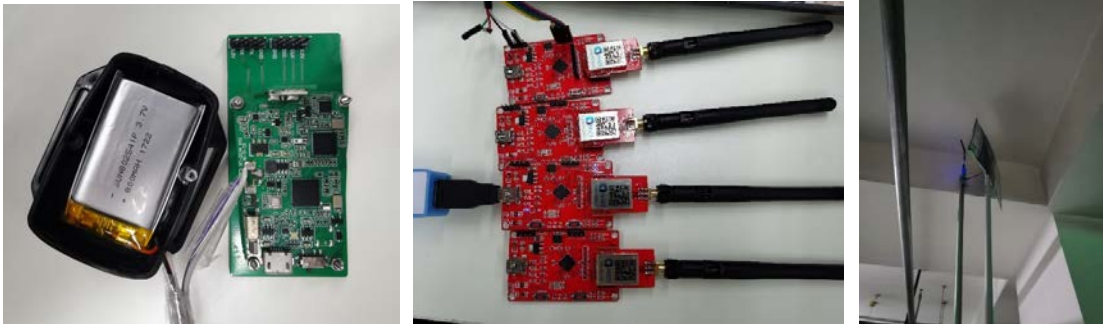


Fig.5 data Collection Equipment

Two typical experimental scenarios are selected for data collection. As shown in Fig.6, Ex 1# and Ex 2# are carried out at underground garage whose height is more than 3m, the new main building of Beihang University on the B2 floor of Block C and B1 floor of Block F respectively. The area of

15m*15m is evenly divided into grids at intervals of 1m, geomagnetic strength is collected at each mesh vertex. Table.1 shows more parameter configurations. In Ex 1# and Ex 2#, BS1(0,7,0) was blocked by paperboard as shown in Fig.5.

Table.1 parameter configuration

parameter	configuration
UWB positioning method	TDOA
Geomagnetic matching method	K-NearestNeighbor (K-NN)
UWB BSs coordinates	BS1(0,7,0), BS2 (4,0,3), BS3 (15,7,0), BS4 (8,15,3)
the origin of coordinates	the bottom left vertex in grid graph

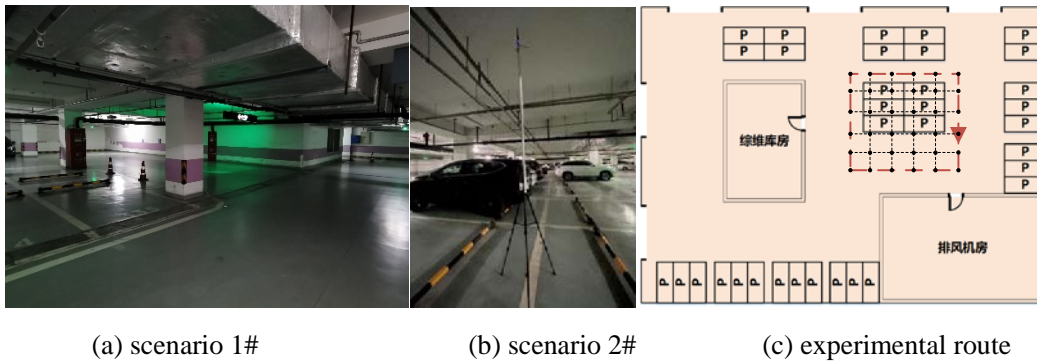


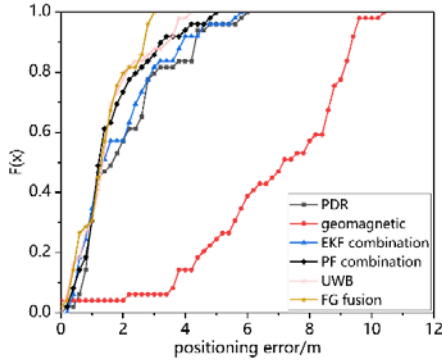
Fig.6 experimental scenarios

For the positioning performance estimation consideration, the mean value, standard deviation of error (STD) and cumulative distribution function (CDF) of positioning error are introduced. Table.2 shows the performance comparison of the positioning

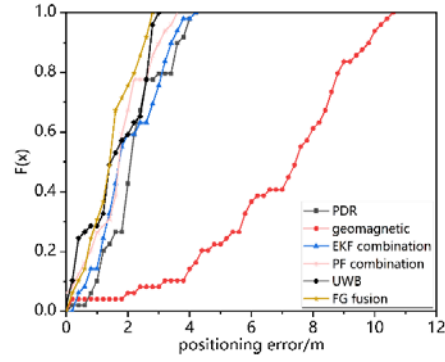
accuracy of different positioning methods and the corresponding CDF curves are shown in Fig.7

Table.2 comparison the accuracy of different positioning methods

Positioning method	mean value/m		STD/m		mean error<2m/%	
	Ex 1#	Ex 2#	Ex 1#	Ex 2#	Ex 1#	Ex 2#
PDR	2.27	2.35	1.46	1.57	53%	33%
geomagnetic	6.94	6.97	2.21	2.58	4%	5%
PF combination	1.80	1.87	1.13	0.96	69%	62%
EKF combination	2.12	2.19	1.32	1.05	56%	57%
UWB	1.64	1.74	1.03	0.99	73%	58%
Factor graph fusion	1.52	1.66	0.98	0.84	79%	71%



Ex 1#

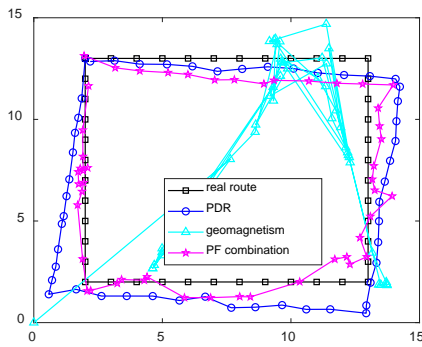


(b) Ex 2#

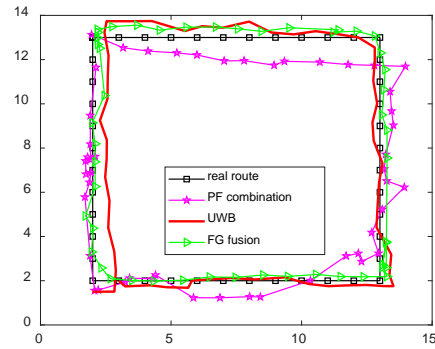
Fig.7 CDF curves of different method positioning error

For the first stage, the improved PF algorithm outperform EKF^[23-24] for PDR/geomagnetism combination from Table.2 and Fig.7. In terms of mean positioning error, the positioning error of improved PF are 1.80m and 1.87m in Ex 1# and Ex 2#, which is lower than that of EKF; in terms of STD of positioning error, PF method is the lowest in all PDR and geomagnetism positioning methods, which indicates its positioning points error has less polarized; in terms of CDF of positioning error, the positioning points within 2m after PF fusion account for 69% and 62% of all reference points, which is reduced by 13% and 5% comparing with EKF. From the trajectory of Ex 1# (a) and Ex 2# (a), it's obvious

that PDR accumulate seriously over time and geomagnetism has bigger error. Improved PF is closer to the real trajectory. In fact, the improved PF combine the characteristics of the fingerprint of geomagnetism and particles in traditional PF. It narrows the particle dispersal range in prediction step compared the traditional particle filter algorithm dramatically; in addition, compared with the fingerprint matching of the entire regional fingerprint database in the entire target area to be located, the improved algorithm reduces the number of matching points with the static geomagnetic fingerprint database, which improves the efficiency of the algorithm compared with the traditional PF.

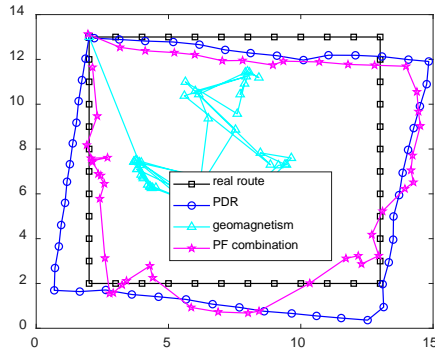


(a) PDR vs geomagnetism, PF

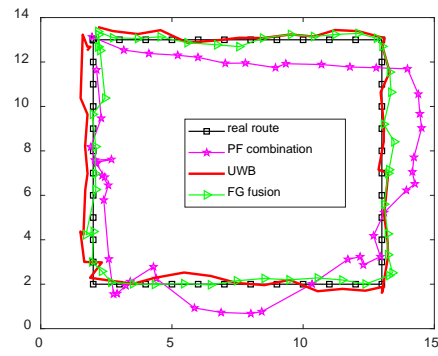


(b) FG vs PF, UWB

Fig.8 trajectory diagram



(a) PDR vs geomagnetism, PF



(b) FG vs PF, UWB

Fig.9 trajectory diagram

For the second stage, FG fusion positioning provides a lower mean positioning error than UWB positioning from Table.2 and Fig.7. In terms of mean positioning error, the mean positioning error of FG fusion positioning method is 1.52m and 1.66m, which is reduced by 7.3% and 5.0% in Ex 1# and Ex 2#; in terms of STD of positioning error, positioning points error of FG has a smaller STD, which indicates there is less hopping points; in terms of CDF of positioning error, reference points of UWB positioning error is within 2m account for 73% and 58%, which is lower than that of FG by 79% and 71%. From the trajectory of Ex 1# and Ex 2#, the positioning accuracy of UWB decreases seriously on the left side of the BS block area and there are some hopping points. However, the trajectory of FG is closer to the real trajectory. Part of hopping points are eliminated obviously. Therefore, FG fusion positioning algorithm mainly solves the problem of UWB positioning hopping points with large positioning error.

IV. Conclusion

In this paper, a two-stage fusion localization model for UWB/PDR/geomagnetism is proposed in the special environment of underground space. PDR/geomagnetism combination positioning is realized by using the improved PF algorithm based on regional constraints, and it is further fused with the UWB localization results by FG. The experimental results show that the proposed

improved PF combination algorithm can effectively improve the positioning accuracy and algorithm efficiency. After FG fusion, the positioning hopping points with error over 2m are decreased and it effectively improves the problem of attenuation of positioning accuracy due to signal occlusion in some areas of UWB. Finally, the proposed two-stage fusion positioning model in this paper can be combined with other mainstream positioning technique according to the actual environment and the need for positioning.

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